A STUDY OF INTERACTIONS OF CARBON NUCLEI IN EMULSION AT 4.5 A GeV/c

ABSTRACT

Ph. D. THESIS

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The present study is based on an experimental analysis of 1300 $^{12}$C-Em collisions at 4.5 A GeV/c. The main aim behind the work is to obtain some useful and important information regarding the mechanism of high energy nucleus-nucleus collisions.

The historical background, energy of the beam, general information on heavy ion collisions, some relevant results on heavy ion collisions and important theoretical models have been discussed. The description of the experimental techniques has been discussed in brief; it gives an idea of nuclear emulsion, classification of tracks, selection criteria, method of measurements and description of various parameters such as range, ionization, delta rays and measurements of angle etc.

The general characteristics of shower, grey, black and heavy particles such as their multiplicities, angular distribution, correlations among secondary particles etc. have been studied. It is found that the average multiplicity of black particles $\langle n_b \rangle$ does not depend on the mass of the projectile. Whereas the average multiplicities of shower and grey particles increase with the mass of the projectile and the dependence can be described by a relation of the type $\langle n \rangle = \text{Const.} \cdot A^q$. The dependence of shower particles' multiplicity on $n_s$ is checked and it is found that the peak of the $n_s$ distribution shifts towards higher value of $n_s$ with increase in $n_g$. The dispersion, multiplicity moment
multiplicity scaling of the shower particles have been studied to check the validity of Koba-Nielsen and Clesen (KNC) scaling in nucleus-nucleus collisions. The main features of the KNC scaling are seen to agree with the data.

The study of correlations between \( \langle n_s \rangle, \langle n_g \rangle, \langle n_b \rangle \) and \( \langle N_h \rangle \) show that these parameters are linearly related to each other. The correlations of the type \( \langle n_i(n_j) \rangle \) \( (n_i, n_j = n_s, n_g, n_b, N_h, i \neq j) \) can be represented satisfactorily by the linear function of the form \( \langle n_i(n_j) \rangle = a_{ij} n_j + b_{ij} \). The results show that the correlations between multiplicities of slow particles, i.e., between grey, black and heavy particles seem to depend on the nature of the incident particle. The shape of the angular distributions of grey and black particles do not depend on the mass of the projectile. This indicates that the production mechanism of heavy particles is probably the same in p-nucleus and nucleus-nucleus collisions. Moreover, these distributions do not exhibit any peaks that could be attributed to the shock-wave phenomenon.

We study central \(^{12}C-\text{Em}\) collisions and results are compared with relevant data from collisions of other projectile with emulsion. The probability of central collisions increases with the mass of the projectile. This result can be explained by the fact that, at high energy, the inelastic cross-section is independent of energy and it increases with the mass of the projectile. For central collisions, the average multiplicity of grey particles,
\[ \langle n_g \rangle, \ \text{increases while that of black particles, } \langle n_b \rangle, \ \text{decreases with the mass of the projectile. This can be explained on the basis of the fireball model. Moreover, it is found that there is strong correlations between the average shower particles' multiplicity, } \langle n_s \rangle \ \text{and the energy available in the centre of mass system, } E. \ \text{The relation } \langle n_s \rangle = -(10.0 \pm 2.2) + (8.1 \pm 0.9) \ln E \text{ gives best fit to the data. The angular distributions of shower, grey and black particles do not depend on the mass of the projectile and the target.}

\text{The method of rapidity gaps has been used to study the cluster formation at the accelerator energy. For this, the rapidities of all the particles of an event are arranged in increasing order } (\eta_1 < \eta_2 < \eta_3 < \ldots < \eta_n). \ \text{The differences } r(2) = \eta_{i+1} - \eta_i \ \text{are calculated. These are called two particles' rapidity gaps. Similarly three and four particles rapidity gap distributions have also been obtained. We observe that in two particles' rapidity gap distributions sharp peaks are observed at small values of rapidity gaps which is an evidence for strong correlations among secondary particles. Thus our results support the idea of cluster formation at this energy. The three and four particles' distributions show that higher order correlations are not present at this energy.}

\text{Finally, our investigation is devoted to the study of fragmentation characteristics of } ^{12}\text{C-nuclei in emulsion at primary momentum } p_0 = 4.5 \ \text{A GeV/c. Multiplicities of projectile fragments}
of different charges have been obtained in different ensembles of collisions. It is found that the average multiplicities of projectile fragments have a weak dependence on the mass of the target. However, the average multiplicity increases with the mass of the projectile. The dependence can well be described by a relation of type: \( \langle N_z \rangle = \text{Const.} \ A^\alpha \).

Our study of the fragmentation of \(^{12}\text{C}\) nuclei in emulsion shows that the principle of projectile fragmentation observed in electronic experiments does not hold under the condition of 4\(\pi\)-geometry. It means that the fragmentation of the projectile nucleus cannot be described in terms of the participant-spectator model. Our data indicate that the principle of factorization has only a limited region of applicability. To test the validity of limiting fragmentation hypothesis, the projected angular distributions and momentum distributions have been studied. The angular distributions of the projectile fragments are typically narrow and their dispersions decrease with increasing fragment charge \(Z\).

The transverse momentum distributions of projectile fragments have also been studied. It is found that the distributions could be described by a Gaussian curve of the type \( N(P) = A \exp \left( -p^2/2\sigma^2 \right) \) in the rest frame of the projectile nucleus and the standard deviation of the distribution has a parabolic dependence on the mass of the fragment. The presence of large \(p_t\) particles distort the transverse momentum distributions. However, for \(p_t \leq 500\ \text{MeV}/c\) the distributions agree with the predictions of the fragmentation...
model. It is also found that the average transverse momentum \( \langle p_t \rangle \) of fragments of different charges produced in collisions of \(^{12}C\) with different target groups increases with target mass.

Azimuthal correlations have also been studied for \( Z > 2 \) fragments. It is found that there exist statistically significant azimuthal correlations among the projectile fragments. This indicates that the fragmenting nucleus gets a transverse momentum during the collision.

Finally, we study the interaction mean free path in nuclear emulsion of fragments of charge \( Z = 2 \) and its possible dependence on distance. We also study the dependence of the mean free path on the distance \( D \) from the interaction vertex for fragments with \( \theta \leq 1^\circ \) and \( \theta > 1^\circ \), where \( \theta \) is the angle of emission of the fragment. We do not find any evidence for anomalously shorter mean free path for \( Z = 2 \) fragment in the first few centimeters of the production point. Further, we do not find any evidence for the production \(^5\)He + \(^6\)Be binary cluster system as recently suggested by Bayman and Tang.
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1988
Certified that the work presented in this thesis is the original work of Mr. M. Qaseem Raza Khan carried out under my supervision.

( Rashid Hasan )
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CHAPTER-I

INTRODUCTION

1.1. Historical Background

Discovery of heavy nuclei in cosmic rays by Frier et al (1) in 1948 provided an opportunity of studying nucleus-nucleus collisions at high energies. In the beginning these studies were aimed at determining the fragmentation cross-sections and interaction mean free paths of nuclei as these data are required for estimating the elemental abundances of cosmic rays at the source and for studying their interstellar propagation mechanism. Although these studies provided some very interesting results (2), extensive studies of nucleus-nucleus collisions could not be pursued partly due to low flux and wide energy range of cosmic ray nuclei and partly due to the fact that at that time even the elementary nucleon-nucleon collision mechanism was not very well understood and therefore nucleus-nucleus collisions were regarded as a messy affair. However, commissioning of heavy ion accelerators, Bevalac at Berkeley with energies up to 2.1 A GeV and Synchrophasotron at Dubna with energies up to 3.7 A GeV, in early seventies revived the interest in the study of nucleus-nucleus or heavy ion collisions. A great deal of interest in high energy heavy ion collisions has also been generated by the fact that high energy
heavy ion beams offer the possibility of studying nuclear properties at points away from the saturation density. During the collision, the nuclei may be compressed to more than their normal density. This compression of nuclei may result in density isomers or quasi-stable states existing at other than normal density (3-12). This idea is basically a very intuitive one. However, when the collisions are examined in the light of what is currently known about nuclear physics, it remains uncertain as to whether significant compressions do actually occur. Nevertheless, the theoretical speculations are sufficiently interesting and an experimental search for evidence of this compression should be made.

The search for exotic phenomena has led to the discovery of neutron rich isotopes far from the nuclear line of stability. Symons et al (13) and Westfall et al (14) have accelerated Ar(N/Z - 1.2) and Ca(N/Z - 1.4) beams and discovered 16 new isotopes in projectile fragments. The isotopes Ne(N/Z - 1.8) and Al(N/Z - 1.7) have been observed for the first time. One expects to learn about many more new isotopes in the analysis of $^{238}$U beam. The lifetime and spectroscopy of isotopes far from the line of stability could be studied with these fragments.

While trying to find new phenomena, experimentalists found that the collision process even under the normal conditions was not well understood. Instead they found that it was very important and interesting to investigate the collision mechanism
itself. In the earlier period, studies of projectile fragmenta-
tion gave us the participant-spectator picture of the colli-
sion (15,16). Participants which are the group of nucleons
which mutually interacted, come from the interaction region
which might have been in a state of high density and high
temperature in an earlier stage of the collision. The specta-
tors are the non interacting parts of the projectile and target.
In momentum space which is the space observed in the experiment,
the fragments from the projectile spectator lie in a peak
centred around the momentum per nucleon of the projectile while
the target fragments lie in another peak centred about the zero
laboratory momentum. The participants are distributed in a wide
range of momentum lying between the two peaks.

Measurements of the participants were at first the single
particle (π, p, d, ....) inclusive cross-sections (17,18). A
variety of models under different dynamical assumptions gave
reasonable fit to the data. They ranged from complete thermaliza-
tion of the participant system to the single hard scattering
of nucleons. On the other hand, the mean free path of nucleons
inside the nucleus is of the same order of magnitude as the size
of the collision region (19-22). Thus, it is hard to believe
that either of these extreme assumptions in the models is valid.
To understand the dynamics of the collision, especially at the
fast stage of the collision, a more detailed analysis of inclusive
cross-sections including mass dependence and projectile energy
dependence is needed. Thus, a number of experiments to measure two particle correlations and semi-inclusive cross-sections have been performed. They showed that the collision mechanism consisted of a mixture of several different mechanisms rather than any single one.

In the following sections, we shall summarize our understanding of heavy ion collisions based on experimental data. The geometrical aspects of the collision will be discussed in relations to the impact parameter, the total and reaction cross-sections and to the average multiplicities in sections 1.3 - 1.7. The fragmentation cross-section will be discussed in section 1.8. In section 1.9, we shall briefly discuss the abnormal states of nuclear matter and in sections 1.10 and 1.11, we shall discuss shock-waves and anomalons as these phenomena could be studied in an emulsion experiment. In section 1.12 various theoretical models which have been put forward to explain the mechanism of heavy ion collisions shall be summarized and finally in section 1.13 we shall explain the aim of the present experiment.

1.2 Energy of the Beam

Before we discuss results on nucleus-nucleus collisions obtained by various workers, we shall first discuss the usefulness of a high energy nuclear beam as compared to low energy one in the studies of mechanism of nucleus-nucleus collisions. At
energies greater than 0.1 GeV per nucleon, the de-Broglie wave length of the incident nucleons of the projectile nucleus is shorter than the typical internucleon distance (\(\sim 1.8 \text{ fm}\)) inside the nucleus. This fact implies that projectile nucleons can recognize the individuality of nucleons inside the target and thus nucleons inside the target are considered to be basic constituents rather than the whole target nucleus itself. This situation is different from the nuclear collisions at energies below 10 MeV/n where the de-Broglie wave length is of the order of 5 to 10 fm, which is comparable to the size of the whole nucleus. In the low energy region the whole target nucleus becomes a basic constituent as seen by the beam and a process in which the whole projectile and target nuclei are involved, such as in the formation of a compound nucleus, could possibly occur. Therefore, high energy nucleus-nucleus (A-A) collisions can be considered as a superposition of nucleon-nucleon (N-N) collisions, and these N-N collisions determine the basic reaction mechanism of nucleus-nucleus (A-A) collisions.

1.3 Impact Parameter

The characteristic features of nucleus-nucleus collisions at relativistic energies depend, from geometrical point of view, on the impact parameter. The collision is determined by the value of the impact parameter and can be divided into three categories: peripheral, quasi central and central collisions, corresponding to large, medium and small values of impact.
parameter respectively. Figure 1.1 shows these three types of collisions quite clearly. If $R_1$ and $R_2$ are the radii of the projectile and target nuclei respectively and $b$ is the impact parameter then

\[ b \simeq (R_1 + R_2) \quad \text{for peripheral collisions} \]
\[ (R_1 + R_2) > b > |R_1 - R_2| \quad \text{for quasi central collisions} \]
and
\[ 0 \leq b < |R_1 - R_2| \quad \text{for central collisions}. \]

In peripheral collisions, the colliding nuclei are well separated in their centres. This allows only a small momentum transfer between the nuclei, leading to the breakup of one or both of them into fragments. The characteristics of the emitted fragments are determined by the intrinsic Fermi momentum distribution of nucleons within the fragmenting nuclei (23). The projectile fragments are emitted within a narrow cone around the beam direction, while the target fragments are nearly isotropically distributed in the laboratory frame. The rapidity distribution consists of projectile and target fragmentation regions which are well separated at relativistic energies, as shown in Fig. 1.1a.

In quasi central collisions, projectile and target nuclei are close to each other while in central collisions they are closer. The difference in the two types could be understood on the basis of number of nucleons taking part in the reaction. In both cases, the whole kinematically allowed rapidity space is
Fig. 1.1 A schematic outline of pseudorapidity distributions in heavy ion collisions at high energy.
available for produced particles, the difference being in the degree of population of the central region. In the central collisions (which are more violent and more complex), we shall expect almost complete extinction of projectile fragmentation products and the rapidity space available for the particles is almost limited to the region between projectile and target fragmentation (Fig. 1.1b and 1.1c).

An important observation (24) about the centrality of collisions needs mention. The cross-section for central collisions, according to the geometrical definition, will be very small when the sizes of colliding nuclei are comparable. In the extreme case, the probability for central collisions will become zero when \( R_1 = R_2 \). This shows that a strict geometrical definition of central collisions is not appropriate. In fact we do not have any strict definition of what we mean by a central collision. Instead, the selection criteria used in the experiments to avoid peripheral collisions determine the centrality.

1.4 **Multiplicity Measurements**

The experimental information from nuclear emulsion experiments is mostly based on the multiplicity and angular distributions of the produced particles. These act as raw material for extracting other vital information. The charged particle multiplicity gives additional information about the geometrical aspect of heavy ion collisions.
The primary aim of these studies is to investigate the dependence of fragment yield on energy transfer between the colliding nuclei and their masses or charges. Experiments involving \( p,d \) and \( ^4\text{He} \) collisions with uranium (25) as well as of \( ^{12}\text{C} \) and \( ^{20}\text{Ne} \) with uranium (26) have shown that \( ^4\text{He} \), \( ^{12}\text{C} \) and \( ^{20}\text{Ne} \) give definitely large fragment yield indicating that nuclei can deposit more energy than the other forms of hadronic probes such as pions or nucleons. Zabelman et al (25) concluded that in \( \alpha \)-particle collisions, the deposition energies are larger than either in proton or deuteron collisions. Although the cross-sections for the production of fragments from uranium are a factor of 1.5 higher with deuterons than with protons, the energy spectra of these fragments are not significantly different.

Sandoval et al (27) and Nagamiya and Morrissey (28) have demonstrated that charge particle multiplicities and cross-sections scale with participant number for inclusive spectra, according to the relation

\[
g = \left( \frac{Z_p A_T^{2/3} + Z_T A_p^{2/3}}{A_p^{1/3} + A_T^{1/3}} \right)^2,
\]

where \( Z \) and \( A \) refer to charge and mass and \( P \) and \( T \) stand for projectile and target respectively. An interesting analysis was done in this regard by Gutbrod et al (29). They showed that average associated multiplicity scaled with the kinetic energy of the projectile for all types of projectiles, where associated means the multiplicity measured when a proton with
energy between 40 to 200 MeV was detected at 90° in the laboratory frame. Angelov et al (30) studied the experimental data on the multiplicity of negative particles produced in collisions of light nuclei p, d, He and 12C at momentum 4.2 A GeV/c with propane and tantalum and they concluded that the average multiplicity depends on the atomic weight of the incident nucleus. Nagamiya et al (17,31) have measured the production of π⁺, π⁻, p, d, ³H, ³He and ⁴He at laboratory angles from 10° to 145° in nuclear emulsion for Ne-NaF, Ne-Cu, and Ne-Pb reactions at 400 A MeV, C-C, C-Pb, Ne-NaF, Ne-Cu, Ne-Pb, Ar-KCl and Ar-Pb reactions at 800 A MeV and Ne-NaF and Ne-Pb reaction at 2.1 A GeV. For equal mass nuclear collisions, the total integrated yields of nuclear charges are well explained by a simple participant-spectator model. The ratio of low energy π⁻ to π⁺ as well as that of ³He to ⁴He, is larger than the neutron to proton ratio of the system. The yield ratio of composite fragments to protons strongly depends on the projectile and target masses and the projectile energy, not on the emission angle of the fragments. A similar result was obtained by Brockmann et al (32) at 1.08 A GeV in (Ar-KCl) collisions. Frankel et al (33) measured the cross-section for producing π⁺ and π⁻ at velocities close to that of the centre of mass in Ar-Ca collisions at 1.05 A GeV. The π⁺ and π⁻ data show a flat plateau around Y_{c.m} = 0. The π⁻/π⁺ ratio of 1.5 ± 0.2 is much lower than the theoretical prediction but
quite consistent with the result of Nagamiya et al (17, 31).

Measurements of charged particle multiplicity distributions in the central rapidity region in p-p and p-α and α-α collisions have been reported by Akesson et al (34). The measured central multiplicity distributions in ultra-relativistic p-α and α-α collisions are well fitted using the p-p cross-sections. Dasaeva et al (35) obtained data from a 2-meter propane bubble chamber irradiated with relativistic nuclei p, d, He and C. The data have been used to investigate the dependence of the average multiplicity of secondary charged particles on various types of produced particles in nucleus-nucleus collisions. Strong dependence of the average multiplicity of charged particles on the mass number of the incident nucleus was observed and the multiplicity in a heavy target Ta increased more rapidly than in a light target C. These results were explained on the basis of a simple geometrical picture of the collision by the increase of the average number of nucleon-nucleon collisions in the colliding nuclei.

Using cosmic ray data, Atwater and Freier (36) studied the meson multiplicity as a function of energy, at energies upto 100 A GeV in nucleus-nucleus collisions in nuclear emulsion. The data show that the variation of multiplicity could be explained in terms of a simple nucleon-nucleon superposition model. They concluded that the multiplicity per interacting nucleon in nucleus-nucleus collisions does not differ significantly from
p–p collisions. I. Otterlund (37) has also compiled the average multiplicity data from LBL–Berkeley (38), Dubna (39,40), and cosmic ray works (41–48) in emulsion. The data show that the average pion multiplicity in relativistic heavy ion collisions, can be factorized into one energy independent part, P, and one dependent part n_0(E) as

\[ n_s = n_0(E) \cdot P^x. \]

Many other workers (49–53) also reported results on multiplicity at different energies and for different projectiles.

1.5 Angular and Momentum/Energy Distribution

Efforts to explain the shape and other features of angular and momentum spectra of emission products go a large way in establishing their production mechanism. For example, projectile fragmentation is expected to cause cluster formation in the emitted fragments. Study of their momentum spectra might then help to know the distribution of particles' momenta inside the projectile before fragmentation occurred and so on. We discuss below the results of such measurements.

Heckman et al (54) have measured the projected angular distribution of Z = 1 and Z = 2 secondaries from projectile fragmentation in the collisions of \(^{12}\text{C},^{14}\text{N}\) and \(^{16}\text{O}\) nuclei with emulsion at 2.1 A GeV. These are found to be Gaussian distributions with standard deviation \(\sigma_{Z=1} = 1.3^\circ -1.5^\circ\), \(\Theta_p \leq 16^\circ\) and
\[ \sigma_{Z=2} = 0.65^0, \Theta_p \leq 1.5^0. \] For \( Z = 1 \) fragments the peak is superposed on a broader distribution with \( \sigma = 7.5^0. \) For \( Z = 2 \) fragments also there is a tail extending to large angles. These observations show that \( Z = 1 \) and \( Z = 2 \) fragments are produced with transverse momentum greater than what is characteristic of the peripheral collisions. The principal conclusion they come to is that the projected angular distributions for both \( Z = 1 \) and \( Z = 2 \) fragments emitted from \( N_h = 0 \) type events in emulsion are in agreement with the single particle inclusive spectra (15). The angular distributions are independent of the projectile and exhibit features of limiting fragmentation (54).

Bhanja et al (55) studied the fragmentation of \(^{14}\)N nuclei at 2.8 A GeV/c and observed that the angular distributions of multiply charged projectile fragments (\( Z = 2 \) to 5) emitted from \( N_h \leq 8 \) (peripheral) collisions are found to be similar to single particle inclusive experiments, in agreement with the characteristics of the limiting fragmentation hypothesis.

Gosset et al (18) have made elaborate studies of inclusive reactions of \(^4\)He and \(^{20}\)Ne projectiles with aluminium and uranium targets at selected incident energies ranging from 0.25 to 2.1 A GeV. They concluded that: (i) The angular distributions are smooth and forward peaked, tending to an evaporation peak at low energies. (ii) Specific isotopes production at fixed energies is independent of projectile mass but depends on target material. (iii) The forward peaking increases with
fragment mass for all beam energies.

The experiments of GSI-LBL-ANL collaboration have shown that light and heavy fragments are produced from different classes of collisions (56). Low energy nuclear fragments \((12 \leq A \leq 140)\) from the bombardment of Au by high energy protons, \(^4\)He and \(^{20}\)Ne are measured to yield information on the breakup of the target nucleus. The energy spectra of fragments from the residues of the violent collisions show little dependence on projectile mass and energy while the angular distributions show more or less forward peaking, depending on the projectile mass and energy.

Heckman et al (57) studied angular correlations between projectile and target fragments emitted from nuclear collisions of \(^{238}\)U nuclei with AgBr nuclei at 0.85 A GeV. Their measurements show that both projectile and target fragments exhibit significant asymmetries in the azimuthal correlations. They also suggest that non-trivial angular correlations may be present.

1.6 Total Cross-sections

Jaros et al (58) have made systematic measurements of nucleus-nucleus total cross-sections for a number of projectile/target/energy combinations of light nuclei, i.e., p, d, \(\alpha\) and C at 0.87 and 2.1 A GeV. Glauber multiple scattering theory (59) has been used to predict accurately nucleon-nucleus (N-A) total cross-sections in the few GeV range. The underlying
motivation was to test the applicability of two alternative formalism, viz., Glauber's multiple scattering theory (60) and Gribov's Regge factorization hypothesis (61). The former is essentially a geometrical theory involving the folding of basic nucleon-nucleon scattering amplitudes, with known nuclear matter distributions. The theory has been extended by Czyz(62) to nucleon-nucleon (N-N) collisions and used to predict total and inelastic cross-sections. It predicts that \( \sigma_T \propto (A_T^{1/3} + A_B^{1/3})^2 \). On the other hand, Gribov's hypothesis (61) of the form

\[
\sigma_T(AA) = \frac{\sigma_T(AB)^2}{\sigma_T(BB)},
\]

predicts that \( \sigma_T(AA) \propto A^{4/3} \). (If we let \( B = \text{nucleon} = P \), and use the fact that \( \sigma_T(PA) \propto A^{2/3} \), we obtain

\[
\sigma_T(AA) \propto A^{4/3}.
\]

Thus, factorization predicts \( A^{4/3} \), while a Glauber approach would predict \( A^{2/3} \), quite different and easily testable.

The experimental data of Jaros et al (58) showed agreement with Glauber theory, indicating thereby that the factorization hypothesis holds predictably at high energies. Nagamiya et al (17) and Frankel et al (33) also made systematic measurements for production cross-section for a number of projectile/target/energy combinations of light nuclei.
1.7 Reaction Cross-section

An empirical expression that has traditionally been used to interpret the data on nucleus-nucleus reaction cross-sections is the Bradt-Peters relation (63)

\[ \sigma_{BT} = \pi r_0^2 \left( \frac{1}{3} A_p^{1/3} + \frac{1}{3} A_T^{1/3} - b \right)^2. \]  (1.1)

Here \( A_p \) and \( A_T \) are the baryon numbers of the projectile and target respectively and \( b \) is an overlap parameter, representing diffuseness and partial transparency of nuclear surface. Relation 1.1 has stood the test of time, although the values of \( r_0 \) and \( b \) have kept changing slightly. Consistent fits to the heavy ion cross-sections data for a variety of target/projectile combinations have been reported (64-68) for \( r_0 \) and \( b \) in the ranges \( 1.15 \leq r_0 \leq 1.45 \) fm and \( 0 \leq b \leq 1.5 \) owing to the fact that \( r_0 \) and \( b \) are coupled.

Heckman et al (54) have measured the mean free paths for \(^4\text{He}, \, ^{12}\text{C}, \, ^{14}\text{N} \) and \(^{16}\text{O} \) nuclei at 2.1 A GeV in nuclear emulsion. By fitting the mean free path data to Karol's soft spheres model (69), they have determined the mean nucleon-nucleon cross-section, which can be accounted by Eq. 1.1 with \( r_0 = 1.36 \) fm and \( b = 1.11 \). Here \( b \) is presumed to be a variable of the type \( b = b_0 \left( A_p^{-1/3} + A_T^{-1/3} \right) \), considering \( b_0 \) to be constant. Experiments of Westfall et al (70) with heavy \(^{56}\text{Fe} \) projectile at 1.88 A GeV and target spread over whole of the periodic table (H, Li, C, S, Cu, Ag, Ta, Pb, U), also confirm the above observations, giving
\( r_0 = 1.47 \pm 0.04 \text{ fm} \) and \( b_0 = 1.12 \pm 0.16 \). Recently, Mangotra et al (67) and Bharti (68) have also measured the mean free paths of \(^{56}\text{Fe-Em}\) at 1.7 A GeV and \(^{40}\text{Ar-Em}\) at 1.8 A GeV respectively and found that the relation of Bradt-Peters (63) is consistent with the experimental data.

1.8 Fragmentation Cross-section

It has already been seen that the projectile and target fragmentation products in peripheral collisions are well separated on the pseudorapidity plot. This implies that one should be able to observe pure projectile or pure target fragmentation reactions. If such a separation could be established, it would be tempting to determine to what extent the concepts of scaling and limiting fragmentation could be applied to such nuclear process.

Bevatron experiments on \( 0^0 \) fragmentation of relativistic heavy ions at \( E = 1.05 \) and 2.1 A GeV have shown that the modes of fragmentation are independent of the mass of the target nucleus (65), a result compatible with the principle of limiting fragmentation. Consequently, the fragmentation cross-section, for the reaction \( B+T \rightarrow F+X \), can be factorized (54) according to

\[
\sigma_{BT}^F = \gamma_B^F \gamma_T^F, \tag{1.2}
\]

where \( \gamma_B^F \) is a function dependent only on the masses of the
beam, B, and fragment, F, while $\gamma_T$, called the target factor, depends exclusively on the target mass. Both $\gamma_B^F$ and $\gamma_T$ are independent of the beam energy. Experimental cross-section fits (65) show that both $\gamma_T \propto A_T^{1/4}$ as well as $A_T^{1/3}$ + constant account for the data quite well.

A straightforward test, usually employed for verifying the limiting fragmentation in peripheral collisions, is to compare the reaction cross-sections for the process like $B + T = F + X$ in the region of fragmentation peaks at well separated energies. Jaros et al (58) found that for $^4$He and $^{12}$C projectile at 0.87 and 2.1 A GeV

$$\frac{\sigma_{BT}(0.87 \text{ A GeV})}{\sigma_{BT}(2.1 \text{ A GeV})} = 1.00 \pm 0.01.$$ 
Detailed experiment of Westfall et al (70) confirms the factorization of $\sigma_{BT}$ but gives slightly different results. They find that $\gamma_T \propto A_T(0.177 \pm 0.010)$, while $A_T^{1/3} + B_T^{1/3}$ cannot account for the data. Exceptions to strict factorization have been observed for fragmentation reactions in hydrogen (65), helium (71) and heavy targets, where single nucleon-stripping is enhanced by coulomb dissociation of projectiles in the virtual photon field of target nucleus (65,72). Moreover, experiment of Chernov et al (73) shows that the factorization of fragmentation cross-sections observed at zero angle has probably a restricted region of applicability. It is broken for the integrated cross-sections and the deviation from simple
fragmentation increases with the mass of the incident nucleus.

Thus, the study of projectile and target fragmentation process gives every indication of being a rich source of information on nuclear structure. There is an accumulation of information indicating that both single particle momentum distributions and higher order correlations play an important role in particle production in the regions kinematically forbidden to free nucleon-nucleon (N-N) collisions. The projectile fragmentation at high energies has proved to be a powerful ally in the production of new exotic nuclei. The area of cosmic rays and astrophysics continues to be aided through measurements of various fragmentation cross-sections.

1.9 Abnormal States

One of the most exciting motivations for the high energy heavy ion physicist is the possibility of studying the nuclear equation of state at high densities, temperatures and pressures (5, 74-78) as well as the search for phase transitions into abnormal superdense states of matter like pion condensates (79,81) density isomers (82) and quark matter (10, 83,84). If large compressions do occur, the exciting possibility exists that new states of nuclear matter may be discovered. These investigations are centred around gaining an understanding of the behaviour of the nuclear equation of state. This equation gives the energy per nucleon as a function of density and temperature. It is
difficult to predict what will occur during the collision. Therefore, even if no new states are discovered, any information which can be obtained about the parameters of this equation will contribute to our understanding of the nature of the nuclear force. On the other hand, there are theoretical speculations which indicate that things might be much more exciting. Specifically, there are predictions that at densities several times the normal, nuclear matter may undergo phase transitions.

One possibility for which there have been several theoretical calculations is the transition to a pion condensate state \( (79b, 85) \). As the nuclear density is increased the energy of the particle-hole excitation states which have the quantum number of the pion, \( J^P = 0^- \), decreases and may become zero at some critical density, \( \rho_{cr} \). Since these particle-hole states behave like bosons and at the critical density, \( \rho_{cr} \), could be produced at no energy cost, these quasi-particles should then condensate out of the vacuum. This represents a phase transition of nuclear matter from its normal 'liquid' state to a spin-isospin lattice \( (86) \). It would probably be a second order phase transition and would manifest itself as a shoulder in a plot of the equation of state. There has not been complete agreement among the calculations as to the value of \( \rho_{cr} \). However, a value of 2-3 times the normal seems to be favoured. Unfortunately, most of the calculations are for infinite nuclear matter at zero temperature. The finite size effects and the
large excitation energies necessarily involved in the compression add many complications and may even inhibit the transition from occurring. Also, even if the condensate exists experimental detection may be extremely difficult. Indeed, theorists have had difficulty in agreeing on a signature. They do agree however that the condensate will not lead to copious production of real pions in the laboratory.

Another speculation is that at sufficiently high densities the nucleons will lose their individual identities. Due to asymptotic freedom, the quarks may act like free particles and the nuclear matter may become a free quark gas (87). Calculations based on the MIT bag model have shown that the energy density of this quark phase should vary with mass density, \( \rho \), as \( \rho^{1/3} \) (83). On the other hand, calculations for baryon matter show a dependence linear with \( \rho \). Once again the critical density at which the transition occurs is model dependent but seems to be \( \approx 10 \) times the normal (88-92).

The behaviour of nuclear matter at high densities is extremely important for astrophysics and cosmology, in addition to its interest from a purely nuclear physics viewpoint. The density at the centre of neutron stars is expected to be 3-4 times the normal density (93). Therefore, if they exist at these densities, pion condensation and quark matter may have important consequences for the properties of these highly compressed stellar objects. It is also important to know what
happened in the early universe which according to the present big bang theory was extremely dense and hot (93). The relativistic nuclear collisions probably provide the only means of simulating these conditions in the laboratory.

1.10 Nuclear Shock-wave Fragmentation

The passage of projectile through a nucleus causes density perturbations and the effect is transmitted through nuclear matter. If the projectile velocity is less than the sound velocity in nuclear matter, the perturbation affects the flow in all directions and the density and pressure are smooth functions of locations. In a supersonic collision, the effect of the strong perturbations in density and pressure propagates only downstream. This allows the formation of shock-waves, characterized by near discontinuities in density, pressure and temperature. Regions of high nuclear density (2-4 times the normal density) and high temperature (T\textasciitilde30–200 MeV), called shock zones, are expected to be created along the direction of propagation of the shock-wave.

The idea that a nuclear shock-wave could be produced when a high energy projectile moves through a nucleus was proposed by Glassgold et al (3). Subsequently, several theoretical models for nuclear shock-waves were suggested. The predicted angular distributions of nuclear matter are different in different shock-wave models. Some models predict comparatively narrow
peaks at a straight angle to a conical shock front (3,4,75-77), whereas other models predict broad forward-peaked distribution (94). However, the common prediction of all these models is the preferential emission of target fragments in the direction perpendicular to the Mach shock front. Therefore, the observed peaks in the angular distributions of the reaction products at forward and at backward angles, which by following their positions and their shift with the energy of the projectile, will be interpreted as signature of nuclear shock-waves.

A number of experiments have been performed to search the shock-waves in heavy ion collisions. The conditions for fully developed shock-waves seem to be best fulfilled in central collisions (95,96). The experiments of Baumgardt et al (97,98) show comparatively sharp peaks in the angular distributions of particles emitted from high multiplicity collisions in the bombardment of AgCl crystals with $^4$He, $^{12}$C, and $^{16}$O. The position of the peak moves with projectile energy from $35^\circ$ at 250 MeV/A to $50^\circ$ at 87 MeV/A. For large energies, the peak disappears, reappearing for 2 A GeV at $75^\circ$ and then shifting to $\approx 50^\circ$ at 4 A GeV. These peaks were interpreted as evidence for the formation of shock-waves. In the inclusive experiment of Poskanzer et al (26), no narrow peaks were found in the angular spectra of $^3$He and $^4$He emitted in $^{16}$O bombardment of Ag and U nuclei at 1.05 A GeV/c. The experiment of Jakobsson et al (99), where $^{16}$O-Em collisions were studied, shows a broad
angular distribution of the target particles, which are centered around $60^\circ$ for $0.2$ A GeV/c and almost isotropic at $2.0$ A GeV/c. The angular distributions are in quantitative agreement with shock-wave calculations. However, they did not observe any narrow peaks, neither in the angular nor in the energy distributions of He nuclei. Further, the experiment of Chernov et al (100) does not show any peaks in the angular distribution. Recently, Ghosh et al (101) studied the correlations among the target fragments and found that there are some short range correlations among the target fragments, which may indicate shock-wave formation in nucleus-nucleus collisions.

In conclusion, there are trends observed in the experimental data that could be attributed to shock-wave fragmentation. However, both the data and the theoretical calculations do, in part, contradict each other. More systematic investigations are in progress. At least, the copious production of intermediate energy, high transverse momentum fragments at angles between $10^\circ$ to $60^\circ$ has been unambiguously demonstrated.

1.11 Anomalons

Sporadic observations in nuclear research emulsion evidencing a short mean free path component (anomalons) among relativistic projectile fragments of heavy nuclei in the cosmic radiation have been reported (102–107). Because of limited statistics, possible systematic uncertainties and the imposibility
of such component within known nuclear physics, these observations were never widely accepted. The situation rapidly changed with the availability of relativistic heavy ion beams from accelerators. Controlled high statistics experiments are possible with such beams and various types of detection schemes may be employed. Consequently a number of attempts have been made to study the anomalons. The results obtained are, however, quite confusing, with some supporting and other refuting the existence of anomalons. Thus, even though there have been many anomalons searches, no definite conclusions can be reached at the present moment. Anomalons, if they exist, should have large geometrical size, long lifetime and appreciable production probability. The anomalons will be discussed in detail in Chapter VI, where we study the anomalous behaviour of $Z = 2$ fragments emitted from $^{12}$C-Em collisions at 4.5 A GeV/c.

1.12 Theoretical Models

Since theory has not been able to indicate what are the experimental signature of the phase transitions, the search for them is extremely complicated and ambiguous. Therefore, before much effort is expended in what may be a futile search, we must first try to discover whether the search is justified. That is, we should learn whether the large compressions needed to achieve these densities actually do occur. Hence, we must study the reaction mechanism and find out how the parameters of the system change during the collisions. A large number of models have been
proposed. These models include the thermal model \((108,109)\), the hydrodynamical model \((94,110-112)\), the cascade model \((113-118)\), the hard spheres model \((119,120)\), the extension of independent particle model \((121)\), the extension of coherent tube model \((122)\), the quark model \((123)\), the independent collision model \((124)\), the multichain model \((125)\), the multichain dual parton model \((126)\), and the statistical bootstrap model \((127)\) and so on. However, none of the models listed above, has so far been able to make compatible quantitative predictions. In addition some specialized models \((111, 128-134)\) have also been developed to explain specific aspects of the collision problem, such as projectile fragmentation, correlations among the produced particles, their energy and angular distributions, clustering, relative frequencies and cross-sections of various types of emitted particles etc. It would be tedious to describe all or even a large fraction of these. Therefore, we shall describe the features of the most important models. These models could be divided into three categories: Thermal models, hydrodynamical models and cascade models. In the following we discuss these models briefly.

1.12.1 Fireball Model

The fireball model \((135,136)\) is perhaps the most intuitively simple and was among the first to give reasonably good agreement with data. It is a macroscopic model, in which a
collision between two heavy ions is described as a two-step process. In a fast \((10^{-23} \text{ sec.})\) primary stage, both the projectile and the target are assumed to make a clean cylindrical cuts through each other (Fig. 1.2). The projectile participants are assumed to transfer all of their momentum to the effective centre of mass system of all the participant nucleons forming a fireball which moves forward in the laboratory at a velocity intermediate between those of the target and the projectile. This picture is called the participant-spectator model or nuclear fireball model and the three regions produced are called the participant region, the beam spectator region and the target spectator region. The energy density in the fireball is extremely high. Consequently, it may be treated as an ideal, relativistic non rotational gas, whose properties may be determined by equilibrium thermodynamics. The fireball subsequently expands isotropically in its own centre of mass with a Maxwellian distribution in energy.

The model basically involves three concepts: geometry, kinematics and thermodynamics. The geometry and kinematics give us the forward velocity and the energy of the participant fireball. The thermodynamics assumes that this energy in the fireball is thermalized and that fireball decays as an ideal gas.

**Collision Geometry:** Suppose that the projectile consists of \(Z_p\) protons and \(N_p\) neutrons \((A_p = Z_p + N_p)\) and that the target
Fig. 1.2 Participants and Spectators. Certain part A overlap with a certain part B; they are participants. Parts $A'$ and $B'$ are the spectators.
nucleus consists of $A_T$ nucleons. Then the geometrical cross-section, $\sigma_G$, is approximately expressed as

$$\sigma_G = \pi r_0^2 \frac{1}{3} \left( \frac{1}{3} A_p + \frac{1}{3} A_T \right)^2,$$

where $r_0 = 1.0-1.2 \text{ fm}$. If a proton inside the projectile hits the target, it is classified as a participant, otherwise it remains as a spectator. An estimate of the average number of participants and spectators can be obtained from Glauber theory (118,137-139). The average number of participant protons from the projectile nucleus is approximately given by $Z_p$ multiplied by the ratio of the target cross-section to $\sigma_G$.

$$<Z_{\text{proj}}^{\text{part}} > \approx \frac{Z_p \pi r_0^2 A_T}{\sigma_G}$$

$$= \frac{Z_p A_T}{\left( \frac{1}{3} A_p + \frac{1}{3} A_T \right)^2}.$$  \hspace{1cm} (1.4)

Similarly, we have

$$<Z_{\text{targ}}^{\text{part}} > \approx \frac{Z_T A_p}{\left( \frac{1}{3} A_p + \frac{1}{3} A_T \right)^2}.$$  \hspace{1cm} (1.5)

The total number of protons assigned to the participant, $Z_{\text{eff}}^{\text{part}}$, is thus given by
\[ Z_{\text{eff}} = \left( \frac{2/3 Z_p A_T + A_p^{1/3} A_T^{1/3}}{(A_p^{1/3} + A_T^{1/3})^2} \right) + \left( \frac{2/3 Z_T A_p + A_T^{1/3} A_p^{1/3}}{(A_p^{1/3} + A_T^{1/3})^2} \right) \]

(1.6)

Similarly, the total number of protons assigned to the projectile spectator and target spectator are respectively given by

\[ Z_{\text{eff}}^{\text{proj/spect}} = Z_p - \langle Z_{\text{proj}} \rangle \]

\[ \approx \frac{Z_p (A_p^{1/3} + 2A_p^{1/3} A_T^{1/3})}{(A_p^{1/3} + A_T^{1/3})^2} \]

(1.7)

\[ Z_{\text{eff}}^{\text{targ/spect}} = Z_T - \langle Z_{\text{targ}} \rangle \]

\[ \approx \frac{Z_T (A_T^{1/3} + 2A_T^{1/3} A_p^{1/3})}{(A_p^{1/3} + A_T^{1/3})^2} \]

(1.8)

The yield of protons from the participant region, \( N_{\text{part proton}} \) is the participant proton number \( Z_{\text{eff}} \), multiplied by the geometrical cross-section, \( \sigma_G \), and is given by
The chief merits of this model are its simplicity and non-involvement of adjustable parameters. It is found that it reproduces the experimental data successfully at low energies (upto 400 MeV) but gives overestimates in the higher energy ranges and lower impact parameters (18,108a). Deviations in the experimental results from the predictions of the model have also been reported by many other workers (17, 31, 32, 56, 140).

1.12.2 Firestreak Model

The nuclear firestreak model (109,136,140,141), a generalization of the fireball model, explicitly includes chemical equilibrium among the hadronic species as well as thermal equilibrium. In this model, the overlapping volume of the colliding nuclei is divided into a series of tubes, parallel to the beam axis. Each projectile tube is assumed to interact only with that target tube which lies directly in its path, leading to the firestreak. Each tube-tube collisions is treated as in the fire ball model assuming thermalization to occur in each of the tube-tube collisions separately. The model describes the production of pions and light composite fragments, as well as the production of protons. Additionally, the firestreak model allows for diffused nuclear surface, replacing the more drastic sharp sphere, clean cut geometry of the fireball. The introduction of this refinement is most
important for the production of light composite fragments while the fireball model fails to describe it. The model seems to fit the experimental data fairly well at 90° or more backward angles, but predicts much higher yields in the forward angles (142). It also shows a good agreement with the shapes of the single particle inclusive cross-sections, but overestimates their magnitudes by a factor of more than two (143). Furthermore, in this model angular momentum is explicitly conserved whereas in the fireball model it is not. The only free parameter in the firestreak model is the freeze-out density below which the hadrons stop interacting.

1.12.3 Hydrodynamical Models

Another type of macroscopic models which have been developed to describe nucleus-nucleus collisions are the Hydrodynamical models (94,144). These models are based on the assumption that the mean free path for interaction is much less than the size of the system. Since the transparency of the nuclei increases with increasing energy, these models should therefore work best at relatively low bombarding energies. In these models, when the target and projectile nuclei collide, they instantaneously merge, coming to equilibrium as a drop of nuclear fluid whose subsequent evolution in time is governed by standard laws of hydrodynamics.

Generally these models consider two nuclear fluids,
the target and the projectile. The behaviour of each of
these fluids is determined by the fluid dynamics, conservation equations for nucleon number, momentum and energy.
Additional terms are introduced into these equations to allow for a coupling of two fluids by means of energy and momentum transfer. In addition, the equation of state is used to obtain a relationship between the pressure and the energy density.

The validity of the fluid dynamics model is subject to three constraints: (i) The system comprising of the two colliding nuclei must contain a large number of degrees of freedom, (ii) the collision must last a sufficiently long time for local equilibrium to occur, and (iii) either the bombarding energy must be low or the interaction strength between the two nuclei must be large. Constraint (iii) ensures that the two nuclei merge instantaneously to form a single fluid.

In the two fluid model (112,145), the reaction is described as the collision between two distinct fluid droplets, originally filled with cold nucleon gas. In view of the finite mean free path, it is assumed that the two nuclei can interpenetrate while retaining their identity. The degree of interpenetration is specified by the geometry. The subsequent expansion and cooling of overlap region is almost instantaneous. The equation of motion consists of a double set of relativistic
Euler equations with an additional force term, giving friction, which is proportional to the relative velocity, and is set up to reproduce the momentum transfer expected on the basis of free nucleon-nucleon cross-sections. The two-fluid model does not explicitly take into account the production of pions and light fragments. The production of pions becomes more important as the bombarding energy increases. At energies below 1 A GeV, the two fluids are supposed to coalesce into a single entity (94) and a non relative approach becomes feasible.

A three-fluid model (146) visualizes the formation of a third fluid - a hot dense fire-cloud, resulting from the intranucleonic collisions in the overlap region of the colliding nuclei. This third component is supposed to consist of scattered nucleons, produced $\Delta$-resonance, $\pi$ and $\rho$-mesons. The three fluids interact mutually by particle collisions. The interpenetration of the colliding nuclei is again specified by geometry while the transmutation of the cold fluids into hot one and the particle production and decay process within the hot component, are governed by hydrochemical equations.

1.12.4 Cascade Model

The cascade model (114,147,148) is a microscopic model which is based on elaborate codes developed for nucleon-nucleus collisions. A characteristic feature of these codes is that the nucleus is represented by a continuum - the Fermi distribution with which the nucleon can interact. A collision partner
with random physical properties is selected from the Fermi distribution and after the collision, whose result is again determined randomly, both partners are considered as cascade particles and allowed to interact further. The Fermi distribution is depleted accordingly. The interactions are assumed to be binary and point-like.

The cascade model approach is radically different from that of the thermal models, since no equation of state is assumed. Instead, the nuclear collision is treated from an entirely microscopic viewpoint. That is, the collision is assumed to be made up of a superposition of individual, binary collisions. The history of each particle is treated by Monte Carlo methods with the probability of scattering on another particle given by the free particle cross-sections. The particle travels on straight line trajectories between the collisions. All phase correlations between nucleons are neglected.

There are three problems with this model; (i) The basic asymmetry of the target and the projectile appears less reasonable for nucleus-nucleus collision, (ii) the assumption of binary collisions taking place at a point in space-time implies a restriction to dilute systems only, where it is usually for three or more nucleons to come into contact simultaneously and (iii) the complexity of calculations and highly expensive computer codes. The results obtained are so numerous that often it becomes difficult to judge which results are real physical
effects and which are consequences of numerical procedures.

One substantial difference between the intranuclear cascade model and other models discussed previously is that thermodynamic equilibrium is not assumed to occur during the collision. In fact, the average number of collisions per nucleon is estimated to be relatively small, 2-3. In this model, as well as in the relativistic nuclear two-fluid dynamics model, nucleon-nucleon scattering cross-sections must be used as input data, which limits predictive capability.

1.12.5 Row-on-Row or Linear Cascade Model

The row-on-row model (118) is a subset of the cascade model. It reduces the full three dimensional cascade problem to one dimension by assuming that one row of nucleons in the projectile scatters off only one row of nucleons in the target. In this model, the projectile and the target are made up of tubes or rows and only straight line collisions between tubes are considered. The key assumption is that each projectile nucleon in a given row interacts only once with target nucleon. The idea is to follow the linear cascade of each projectile nucleon separately, which is simulated through computer codes. The model takes transverse communication and thus thermalization cannot take place. This type of model cannot be more valid than a full scale cascade calculation. Its main virtue is that the complexity of the computer code is considerably reduced.
1.13 **Aim of the Present Experiment**

The main aim of the present experiment is to study the general characteristics of $^{12}$C-Em collisions at 4.5 $\text{A GeV/c}$. Using a sample of 1300 events, multiplicity and angular distributions of charged secondaries and correlations among various multiplicity parameters have been studied. A comparison of these results with similar results from p-Em and $\alpha$-Em collisions at nearly the same momentum provides information about the mechanism of particle production in $^{12}$C-Em collisions. In order to find out whether the high energy concept of scaling is applicable to heavy ion collisions also, we study the multiplicity distributions of shower particles.

We also examine the phenomenology of central collisions and investigate those characteristic features of central collisions which might provide some additional insight into the detailed nature of the collision mechanism. The present study represents the most central collisions which have ever been studied and consequently these results are of special interest.

Beside these, the study of multiparticle production phenomenon is also interesting and considerable efforts have been put forward to understand the mechanism of multiparticle production during the recent years. The idea of cluster formation in the intermediate stage of the multiparticle production in high energy collisions has attained wide acceptance. Some information about the cluster can be experimentally obtained
by the study of rapidity gap correlations of the final state particles. Therefore we study the correlations among the secondary particles produced in $^{12}$C-Em and central $^{12}$C-Em collisions at 4.5 A GeV/c.

Another aim of the present experiment is to study the fragmentation of carbon nuclei in nuclear emulsion. The study of this phenomenon would give information about the internal structure of nuclei under condition of small transfer of momentum and energy. The study of fragmentation phenomenon at increasing energies and with heavier projectiles, although becoming more complex, is expected to provide better test for different models. We also test the validity of limiting fragmentation hypothesis. Recently, contradictory results have been obtained about the anomalous behaviour of projectile fragments. In view of this situation, it would be worthwhile to study the anomalous behaviour of projectile fragments. We therefore study the dependence of the mean free path of $Z = 2$ fragments on the distance from the interaction vertex.
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CHAPTER-II

EXPERIMENTAL TECHNIQUE

2.1 Introduction

The study of hadron-nucleus and nucleus-nucleus collisions at high energies has been carried out using mainly the nuclear emulsion. The competitive capabilities of nuclear emulsion are modest in comparison to bubble chamber and counters for studying hadron-nucleus and nucleus-nucleus collisions. However, the nuclear emulsion is a versatile instrument for detecting charged particles. It is not only capable of counting particles, but also giving information concerning their mass, their energy and their modes of collision and decay. The nuclear emulsion has high density and high stopping power about 1700 times the stopping power of standard air. After the development, the nuclear emulsion stacks are kept under specified conditions, and thus the photographed events can be preserved for many years. With the addition of high resolution tracking, computer aided scanning has substantially enhanced the analysing power of this technique.

The drawback with nuclear emulsion is that it requires a special dark room processing and very careful handling before development. The emulsion technique is very slow. Further, it is not possible to predict accurately the target nucleus involved in the collision and the time of particle detection.
The composition of the nuclear emulsion is heterogeneous. It consists of three basic components: (a) Silver halide, mainly bromide, with small admixture of iodine, (b) gelatine and glycerine, and (c) water. The glycerine is used as plasticizer to prevent it from breaking.

The percentage composition of the above is such that about 71% of the collisions occur with heavy nuclei, AgBr, 25% due to light nuclei, CNO and only 4% with hydrogen nuclei. However, the cross-sections of reactions with AgBr, CNO or H nuclei, depend on the mass and energy of the beam particles (1).

When a charged particle passes through emulsion, it loses energy by electromagnetic collisions. The energy lost by the charged particle is transferred to the atomic electrons. As a result of this, the later acquires an excited state. If the energy gained by the electron is greater than the ionization potential, the electron is liberated and the atom is said to be ionized. The ionization of the atom converts some of the halide grains in such a way that they, when immersed in a reducing bath, known as developer, get converted into silver grains which may easily be distinguished because of their black colour. Thus, a series of grains is formed along the trajectory of the particle which is termed as its track. The characteristics of the track are based on the nature and the velocity of the charged particle due to which the track has been caused, e.g., higher the velocity rarer will be the grains formed by the particle and vice versa.
When an energetic particle collides with a target nucleus, a large number of secondary particles, both charged and neutral, come out of it. The charged particles produce their tracks and are thus recorded. The event is called 'Star' because it looks like a star. The slow, medium and energetic particles are characterized by the ionization that they produce in passing through the emulsion. The track of a particle represents its various characteristics. The observable properties of a track are its range, ionization, scattering, delta rays etc.

When a particle of charge $z e$ and mass $M$ traverses a medium of atomic number $Z$ and mass number $A$, it excites and ionizes the atoms of the medium through coulomb interactions. This results in loss of energy of the incident particle. The rate of energy loss $dE$ per unit length $dx$ traversed is given by

$$-rac{dE}{dx} = \frac{4\pi NZ z^2 e^4}{m_e v^2 A} \left[\ln\left(\frac{2m_e v^2}{I(1-\beta^2)}\right) - \beta^2\right], \quad (2.1)$$

where $z e$ is the charge and $v$ is the velocity of the incident particle, $N$ is the number of atoms per cm$^3$ of the stopping material, $Z$ and $I$ represent their atomic number and mean ionization potential respectively, $m$ is the electron mass and $\beta = v/c$. It is clear from Eq. 2.1 that the energy loss does not depend on the mass $M$ of the incident particle. It is only a function of its velocity and charge. Since the logarithmic term varies only slightly with $v$, the energy loss is proportional to
\[ \frac{z^2}{v^2} \text{ and } \frac{Z}{A} \text{ to an approximation at low velocities } (v \ll c). \]

2.2 **Experimental Details**

In the present investigation, an emulsion stack comprising of 40 pellicles of BR-2 emulsion of standard composition, each of dimensions \(18.7 \times 9.7 \times 0.06 \text{ cm}^3\), with printed grid, has been used. The pellicles were tangentially irradiated with 4.5 A GeV/c \(^{12}\text{C}\) beam at the Synchrophasatron of the Joint Institute for Nuclear Research, Dubna, (U.S.S.R).

2.3 **Scanning**

The process of searching the position of the collision in the emulsion pellicles is called the 'scanning of events'. There are two types of scanning: (i) Area scanning and (ii) line scanning.

2.3.1 **Area Scanning**

Area scanning of a pellicle is usually done in strips of width equal to one side of an inscribed square in the field of view. During the scanning, the full depth of the pellicle is examined by rolling the fine focus of control (Z-motion of the microscope). While observing the layers of the emulsion one continually goes on looking for events present in the field of view. One such elementary motion from air to glass side surface of the emulsion is called a 'scanning traverse'. Before shifting
the field more than one traverse is made along the Z-motion. The field of view is then shifted along the X-motion of the microscope until the whole X-strip of the pellicle is completed. On completing the X-strip one switches on to the next X-strip by giving the displacement in Y-direction equal to or less than one field of view. Similarly the whole area of the pellicle is scanned out.

This method is considerably faster than the line scanning, but barring a few special cases the efficiency for finding events with small \( N_h \) and \( n_s \) is poor.

2.3.2 Line scanning

When the emulsion stack is exposed to a parallel beam of particles, nearly parallel to the surface of the emulsion such that the beam particles enter from one emulsion edge (called the leading edge), perpendicular to it and leave the opposite side edge of the emulsion, the line scanning is carried out. In this method a primary track is picked up on the scan line as it enters the stack. The track is examined to ensure that it does not interact before the scan line. The primaries are followed until they interact or leave the pellicle.

The line scanning is effective in the following conditions of exposure: (i) The flux of beam is not dense and is spread out throughout the leading edge. (ii) The available length for the traversal of the beam is large. (iii) The beam does not dip much,
i.e., it traverse a considerable length of pellicle.

For the present study, we have adopted the technique of line scanning as it has more relative efficiency. The pellicles were scanned using Nikon and Cooke M 4000 series microscopes with the following optics: 15 x 40 for scanning and 15 x 100 (oil immersion objective) for measurements. The efficiency of the line scanning is nearly 100% and we picked up almost all events having a difference between the charges of projectile and the principal projectile fragments of $\Delta Z = Z_p - Z_f < 2$. Also, we minimized as much as possible the scanning of beam tracks of $Z_p \leq 5$. However, the scanned beam tracks were further examined by measuring the $\delta$-ray density on each of them. The negligible fraction of beam particles having $Z_p \leq 5$ was thus identified and excluded. The one prong events with an emission angle of secondary particle track $\Theta < 3^\circ$ and without visible tracks from excitation or disintegration of the incident particle and/or target nucleus, were excluded as due to elastic scattering.

Events satisfying the following criteria were selected for final measurements.

(i) The beam track must lie within $\sim 2^\circ$ to its mean direction in the pellicle.

(ii) The event must be 3 mm away from edges of pellicles, so that the possible distortion effects are avoided.

(iii) To facilitate the measurements, the events which were produced within 30 $\mu$m from the top or bottom surfaces of the pellicles have been excluded from the data.
(iv) The colliding primaries were followed back up to the edge to ensure that the events chosen were not due to secondary collisions.

2.4 Classification of Tracks

The secondary particles were classified into different categories according to the following criteria.

(i) Shower(s) particles with relative ionization $g/g_0 < 1.4$, were $g_0$ is the plateau ionization.

(ii) Grey (g) particles with a range in emulsion $L \geq 3$ mm and $1.4 \leq g/g_0 < 10$ and having a dip angle $\theta_d < 30^\circ$.

(iii) Black (b) particles with a range in emulsion $L < 3$ mm and $g/g_0 \geq 10$ and having dip angle $\theta_d < 30^\circ$.

The grey and black particles are collectively called the heavy particles, i.e., $N_h = n_g + n_b$. To take into account the grey and black particles with $\theta_d < 30^\circ$, a geometrical weight factor $W$ was attached to each grey and black particle such that

$$W = \begin{cases} 1 & \text{when } 150^\circ \leq \theta \leq 30^\circ, \\ \frac{\pi}{2 \sin^{-1} \left( \frac{\sin 30^\circ}{\sin \theta} \right)} & \text{otherwise} \end{cases}$$

where $\theta$ is the space angle.

(iv) Doubly charged fragments ($Z = 2$) of the projectile are the particles with $g/g_0 \approx 4$ and without any change in ionization along a length of at least 2 cm from the interaction vertex and having an angle of emission $\theta < 3^\circ$. 
Multiply charged fragments \( (Z \geq 3) \) of the projectile are the particles with \( g/g_0 > 6, \Theta < 3^\circ \) and without any change in ionization along a length of at least 1 cm from the vertex.

2.5 Track Parameters and Their Measurements

The track parameters which are used for the identification of a particle and estimation of its energy are given below.

2.5.1 Range

The average distance traversed by a charged particle in a medium before its kinetic energy reduces to zero is called the range of the particle. If the distance is measured from an arbitrary point to the stopping point along the track, it is called the residual range. It is a measure of the energy of the particle at that point. The distance traversed by a charged particle in the unprocessed emulsion, with initial kinetic energy \( E_0 \), before coming to rest is (2).

\[
R = \int_0^E \frac{dE}{(-dE/dx)}, \quad (2.3)
\]

where \( dE/dx \) represents the rate of energy loss of the particle.

2.5.2 Ionization

The ionization caused by a particle may be estimated by measuring one of the following quantities on the track of the particle. (i) Grain density, (ii) Blob density, (iii) Blob and
Gap densities (iv) Integral gap length, (v) Mean gap length, (vi) Delta rays and (vii) Track width. However, it is found that all the methods have certain limitations and none is applicable to all types of tracks. In the following sections we discuss only those methods which have been used for the identification of the particle in our experiment.

2.5.2.1 Grain Density

The track of a particle in emulsion appears as minute trail of silver grains. The number of developed grains per unit path length, termed as the grain density, is found to be a reliable parameter for estimating the ionization caused by the particle. However, the grain density, \( g \), in a track corresponding to a particular value of ionization depends on the degree of development of the emulsion. For accurate results, it is therefore necessary to determine the ratio, \( g^* \), of the observed grain density, \( g \), to the corresponding value, \( g_0 \), on the track of any other particle of charge \( e \) moving in the same emulsion with relativistic velocity. The normalization is made by choosing the comparison track of the relativistic particle in the same region of the emulsion.

2.5.2.2 Blob Density

If the velocity of a particle is not high, some of the grains in the track are clogged together to form a blob. The
grain counting on such track is difficult because the true number of grains is uncertain. In such cases, the number of individually resolved grains, or groups of grains, is counted without discriminating between them or attempting to estimate the number of grains in the clusters. It can be applied for a limited range of values of the ionization. The value of ionization in such cases is obtained by the following expression:

$$B = g \exp (-\alpha \cdot \sigma).$$  \hspace{1cm} (2.4)

When ionization is determined by blob counts alone, the statistical error in the measurement is calculated from (2).

$$\frac{dg}{g} \approx \frac{1}{\sqrt{N_B}} \frac{\exp (-\alpha \cdot g)}{(1 - g\alpha)}. \hspace{1cm} (2.5)$$

In fact for low values of $g$ the above relation would tend to have the form

$$\frac{dg}{g} \approx \frac{1}{\sqrt{N_B}}. \hspace{1cm} (2.6)$$

2.5.2.3 Blob and Gap Densities

Fowler and Perkins (3) have used a method to estimate the ionization from the ratio of the number of observable blobs to the number of gaps. The space between two adjacent grains is called the gap. We select a particular length of the track which is to be classified and count the number of gaps, $H$ and
the number of blobs, \( B \), in the length. The length is adjusted in such a way that \( B \propto 4H \). The identification of the charge of a secondary particle is done by taking into account the blob density (\( B \)), the density (\( H \)) of the gaps greater than a length (\( t \)) 1.1 \( \mu \)m and the gap length coefficient (\( G \)), which is given by (4)

\[
G = \frac{1}{t} \ln(B/H).
\]  (2.7)

2.6 Delta-ray Density

When the energy of an ionized electron is sufficient to give rise a recognizable track, the track is called the \( \delta \)-ray. The track is generally recognizable if it has four or more grains. The \( \delta \)-ray density, i.e., the number of \( \delta \)-rays per unit track length depends on the convention adopted and also on the charge and velocity of the moving particle. If \( \nu \) represents the \( \delta \)-ray density for a singly charged particle having velocity \( \beta v \), the density of other particles \( n_\delta \) is given by

\[
n_\delta = Z^2 \nu, \quad (2.8)
\]

where \( Z^2 \) is the mean square effective charge for \( \delta \)-ray production.

The total number of \( \delta \)-rays over the velocity interval 0 to \( \beta v \) is given by
\[ N_\delta = \int_0^R n_\delta \, dR, \quad (2.9) \]

where \( R \) is the range of the particle having velocity \( \beta c \) and the quantity \( N_\delta \) is known as the integral number of \( \delta \)-rays.

We selected a forward cone for the identification of charge of all projectile fragments with \( Z \geq 2 \). All grey looking tracks traversing more than 2 mm in the stack and lying within a cone of \( 3^\circ \) were selected for charge identification. The \( Z = 2 \) fragments are easily identified by their grain densities which are four times the density of a minimum ionizing track and do not change upto a distance of \( \sim 2 \) cm from the interaction vertex. We also used \( \delta \)-ray count method for identifying the \( Z = 2 \) fragments. Events in which the projectile carbon was dissociated into three \( \alpha \)-particles (\( ^{12}C \rightarrow 3\alpha \)) were used for charge calibration.

For the identification of fragments with \( Z \geq 3 \), we counted the number of \( \delta \)-rays. Charge calibration was done on tracks of beam nuclei and \( Z = 2 \) fragments from \( ^{12}C \rightarrow 3\alpha \) events. To check the charge identification, the charges of half of the fragments were also estimated by measuring the blob density (\( B \)), density (\( H \)) of gap greater than 1.1 \( \mu \)m and the gap length coefficient \( G \), (Eq. 2.7). For calibration, tracks of beam nuclei and the \( Z = 2 \) fragments from \( ^{12}C \rightarrow 3\alpha \) were used. The results of the two methods were in good agreement.
2.7 **Anole Measurements**

2.7.1 **Projected Angle**

To measure the space angle of a track with respect to the primary, its projected angle in X-Y plane with respect to the X-direction is measured. It can directly be measured by a goniometer having a least count of $0.25^\circ$ under a high magnification. One of the eyepieces of microscope is replaced by the goniometer. The vertex of the collision is focussed at the centre of the goniometer. The primary beam track is aligned with one of the reference line of the goniometer. Now secondary tracks are aligned one by one with the other reference line and the goniometer scale reading is taken for the projected angle with respect to the forward direction of the primary beam.

2.7.2 **Dip Angle**

In the processed emulsion, if $\Delta Z$ is the difference between $Z$-coordinates at two points on a track separated by a distance $\Delta X$, the angle

$$\Theta_d = \tan^{-1} \left( \frac{\Delta Z}{\Delta X} \right),$$

is called the dip of that part of the track. The dip in the unprocessed emulsion may then be given as

$$\Theta_d = \tan^{-1} \left( \frac{S \cdot F \cdot x \cdot \Delta Z}{\Delta X} \right),$$
where S.F is the shrinkage factor of the emulsion. Thus, the dip angle of a track is calculated by measuring the Z-coordinates of two points on the track separated by a known distance.

2.7.3 Space Angle

Once we know the dip angle and projected angle with respect to X-axis in the XY plane, the space angle of a track can be calculated using the expression

\[ \theta_s = \cos^{-1} [\cos \theta_p \times \cos \theta_d]. \] (2.12)

When the angular separation between the tracks in the forward cone is very small, it is difficult to measure the projected and dip angles due to overlapping. In such cases, the X,Y,Z coordinates have to be measured. To measure the angle, first the beam track of the star is aligned parallel to the X-motion of the microscope. The vertex of the star is focussed and the reading of Z-coordinate is taken. Now the stage is moved forward to atleast five fields of view. Again, a point on the beam track is focussed and the Z reading gives the \( \Delta Z \) reading for the projected length \( \Delta X \). The number of fields of view shifted gives the \( \Delta X \) reading for measured \( \Delta Z \) reading. \( \Delta Y \) reading is taken from the eyepiece graticule scale for a segment \( \Delta X \). Similarly, the \( \Delta Y \) and \( \Delta Z \) readings for each track of the star are taken. While taking the Y and Z reading care is taken of directions. The projected and dip angles are
given respectively by

\[ \theta_p = \tan^{-1} \left( \frac{\Delta Y}{\Delta X} \right), \quad (2.13) \]

and

\[ \theta_d = \tan^{-1} \left( \frac{S \cdot F \cdot x \Delta Z}{\Delta X} \right). \quad (2.14) \]

The space angle is then determined using the relation 2.12.
REFERENCES

CHAPTER-III

GENERAL CHARACTERISTICS OF $^{12}$C-Em COLLISIONS AT 4.5 A GeV/c

3.1 Introduction

The acceleration of heavy nuclei to relativistic energies opened up a new area in the field of heavy ion physics. The goal of this new field is to discover the quark-gluon plasma. However, in order to detect this abnormal nuclear phenomenon, first the normal nuclear collision mechanism has to be understood. For this measurements on multiplicity, correlations between various multiplicity parameters, angular distribution of various particles etc. are needed. Therefore, in this Chapter, our aim is to study the general characteristic features of $^{12}$C-Em collisions at 4.5 A GeV/c. Using a sample of 1300 events, multiplicity distributions of shower, grey, black and heavily ionizing particles and correlations among various multiplicity parameters have been studied. The dispersion, multiplicity moments, multiplicity scaling of the shower particles have been studied in order to check the validity of KNO scaling in nucleus-nucleus collisions. Angular distributions of charged secondaries and target fragments have also been studied. The results obtained are compared with the data at nearly the same incident momentum per nucleon from proton-nucleus and nucleus-nucleus collisions in order to trace the dependence of various parameters on the projectile and target mass.
3.2 Experimental Results

3.2.1 Mean Free Path

A total of 3065 inelastic collisions of carbon nuclei were recorded by following 42269 cm of primary track length, leading to the mean free path of 4.5 A GeV/c carbon nuclei in emulsion $\lambda = (13.79 \pm 0.25)$ cm. Out of these, 1300 collisions were finally picked up, without any bias, for the final analysis. The details of scanning, classification of tracks and measurements have already been given in Chapter II.

One can deduce the following relationship between the charge $Z$ of a projectile and its mean free path $\lambda_z$, from the prediction of Bradt-Peters relation (1) with appropriate assumptions:

$$\lambda_z = \Lambda Z^{-b}$$

(3.1)

Here $\lambda_z$ is the mean free path of the projectile of charge $Z$ and $\Lambda$ is the charge independent mean free path. Figure 3.1 shows the experimental values of the mean free path of various projectiles in nuclear emulsion. Data points for the projectiles $^1$H, $^4$He, $^{14}$N, $^{16}$O and $^{56}$Fe have been taken from the works of Chernov et al (2) and Heckman et al (3) while that for $^{12}$C from the present work. The experimental data fit well with Eq. 3.1 for $\Lambda = 28.0 \pm 0.70$ and $b = 0.39 \pm 0.02$. These values are in agreement with those reported by other workers (4,5). $\Lambda$ and $b$
can also be estimated by calculating the cross-sections using the overlap formula together with the information about the isotope production cross-section or a parabolic mass distribution centred around the most stable isotope.

3.2.2 Target Identification

For a qualitative verification of the hypothesis of factorization of cross-section, we carried out target identification for our \(^{12}\text{C-Em}\) collisions. The exact identification of target in an emulsion experiment is not easy as the medium is composed of H, C, N, O, Ag and Br nuclei. Statistically the classification of collisions with different target nuclei in emulsion could be done on the basis of the distribution of heavy particles \(N_h\), which is a characteristic of the size of the target. Generally, events with \(N_h \leq 1\), \(2 \leq N_h \leq 7\) and \(N_h \geq 8\) are classified as collisions with hydrogen (H), light nuclei (CNO) and heavy nuclei (AgBr) respectively.

Barashenkov et al (6a) and Jakobsson and Kullberg (6b) have studied the distribution of short range tracks in p-Em and \(^{16}\text{O-Em}\) Collisions. They observed that in AgBr events with \(N_h < 8\), there are no tracks with range \(R\) between \((10-50) \mu m\). Moreover, their observation indicate that the Coulomb barrier is high enough to depress the emission of low energy light particles \((H + He)\) only in peripheral collisions with a heavy target and therefore the presence of short range tracks
(R ≤ 10 μm) in peripheral collisions are due to recoil nuclei.

In view of the above consideration, we have used the following criteria for target identification.

AgBr events: \( N_h > 7 \)

or

\( N_h ≤ 7 \) and at least one track with \( R ≤ 10 \) μm and no track with \( 10 < R ≤ 50 \) μm.

CNO events: \( 2 ≤ N_h ≤ 7 \) and no track with \( R ≤ 10 \) μm.

H events: \( N_h = 0 \)

or

\( N_h = 1 \) but not falling in any of the above categories.

In Table 3.1 we present the percentage of different kinds of events in collisions induced by various projectiles in emulsion.

3.2.3 Multiplicity Distributions of Secondary Particles

When a high energy projectile hits a nucleus, a number of charged and uncharged particles are produced. The emergence of fast particles producing showers and grey tracks in nuclear emulsion occurs in a very short time after the instant of impact of the projectile. Thereafter the nucleus remains in an excited state for a quite long time on nuclear time scale. Finally, the
nucleus de-excites resulting in the emission of large number of nucleons and other heavy fragments. This process is known as the evaporation process. The particles emitted through this process generally appear as black tracks in emulsion.

Table 3.2 presents the mean multiplicities of shower, grey and black particles from $^{12}$C-Em collisions. Also present in the table are similar results from collisions of various projectiles with emulsion. It can be seen that $\langle n_b \rangle$ remains practically unchanged for different projectiles ranging from proton to iron, indicating the approximate equality of the residual nucleus excitation. A comparison of the data presented in Table 3.2 shows that $\langle n_g \rangle$ increases with the mass of projectile. This can be explained in terms of the fireball model (7). According to the model, the grey particles come from the participant volume and the number of participant nucleons increases as the volume of the cylinder cut in the target by the projectile increases. This volume increases with the increase in the mass of the projectile and consequently the average number of grey particles increases. Figure 3.2 shows the dependence of $\langle n_g \rangle$ on the mass of projectile. The dependence can be described by a relation of the type: $\langle n_g \rangle = \text{Const.} \, A^\alpha$, where the best fit value of $\alpha$ is $(0.34 \pm 0.11)$.

The average values of the multiplicities of charged particles in different ensembles of p-Em (8), $^{12}$C-Em and $^{56}$Fe-Em(9) are given in Table 3.3. From the table it is
Fig. 3.1 Interaction mean free path, $\lambda_z$, of various projectiles in emulsion versus charge, $Z$, of the projectile. The solid line represents the best fit to the data (Eq. 3.1 of the text).

Fig. 3.2 Mass of the projectile versus $\Lambda_{\text{BHV}}$. The average multiplicity of gray particles as a function of the mass of the projectile, $\Lambda$. The solid line shows the relation $\langle n_g \rangle = \text{Const.} \Lambda^\alpha$. (See Text.)
Table 3.1
Percentage of occurrence of collisions with various groups of nuclei in emulsion

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Momentum (A GeV/c)</th>
<th>Target</th>
<th>H</th>
<th>CNO</th>
<th>AgBr</th>
<th>Total no. of events</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>702</td>
</tr>
<tr>
<td>α</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1100</td>
</tr>
<tr>
<td>C</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1300</td>
</tr>
<tr>
<td>O</td>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>269</td>
</tr>
<tr>
<td>N</td>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>923</td>
</tr>
<tr>
<td>Fe</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>935</td>
</tr>
</tbody>
</table>

Table 3.2
Average multiplicities of secondary particles in p-nucleus and nucleus-nucleus collisions

<table>
<thead>
<tr>
<th>Collision</th>
<th>Momentum (A GeV/c)</th>
<th>(&lt;n_s&gt;)</th>
<th>(&lt;n_g&gt;)</th>
<th>(&lt;n_p&gt;)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-Em</td>
<td>4.5</td>
<td>1.63 ± 0.02</td>
<td>2.81 ± 0.06</td>
<td>3.77 ± 0.08</td>
<td>41</td>
</tr>
<tr>
<td>d-Em</td>
<td>4.7</td>
<td>3.00 ± 0.10</td>
<td>2.30 ± 0.10</td>
<td>5.30 ± 0.10</td>
<td>49</td>
</tr>
<tr>
<td>α-Em</td>
<td>4.5</td>
<td>3.90 ± 0.10</td>
<td>4.70 ± 0.20</td>
<td>4.70 ± 0.20</td>
<td>33</td>
</tr>
<tr>
<td>C-Em</td>
<td>4.5</td>
<td>7.71 ± 0.23</td>
<td>6.05 ± 0.18</td>
<td>4.59 ± 0.14</td>
<td>9</td>
</tr>
<tr>
<td>N-Em</td>
<td>2.1</td>
<td>8.85 ± 0.28</td>
<td>5.29 ± 0.31</td>
<td>4.57 ± 0.21</td>
<td>2</td>
</tr>
<tr>
<td>O-Em</td>
<td>4.5</td>
<td>10.50 ± 0.60</td>
<td>7.60 ± 0.60</td>
<td>4.88 ± 0.29</td>
<td>50</td>
</tr>
<tr>
<td>Fe-Em</td>
<td>2.5</td>
<td>13.30 ± 0.40</td>
<td>8.71 ± 0.34</td>
<td>4.45 ± 0.14</td>
<td>9</td>
</tr>
</tbody>
</table>
observed that with increasing \( N_h \) (Number of heavily ionizing particles) i.e., with increasing target mass, there is an increase in the average shower particle multiplicity, \( \langle n_s \rangle \), in the case of \(^{12}\text{C-Em}\) and \(^{56}\text{Fe-Em}\) collisions, but in the case of \( p-\text{Em} \), it is found to be decreasing. This decrease is argued as being due to the absence of visible meson formation in the secondary process as well the knocking out of the relativistic particles (created) with increasing impact parameter after scattering (8). The average multiplicity of grey particles increases rapidly with increasing \( N_h \) in the case of \(^{12}\text{C-Em}\) and \(^{56}\text{Fe-Em}\) collisions, whereas the average multiplicity of black particles does not seem to increase so rapidly. Figure 3.3 shows the multiplicity distributions of shower, grey, black and heavy particles from \(^{12}\text{C-Em}\) collisions at 4.5 A GeV/c together with those from \( p-\text{Em} \) and \( \alpha-\text{Em} \) collisions at 3.0 GeV/c and 2.1 A GeV/c respectively (10,11). From these figures it follows that:

(i) The \( n_s \) distribution changes most strongly with the increase in projectile mass; its broadness substantially changes its shape. The contribution from small values of \( n_s \) decreases as the projectile mass increases.

(ii) The \( n_g \) distribution of \(^{12}\text{C-Em}\) collisions has a tail up to 41 and differs significantly from those for \( p-\text{Em} \) and \( \alpha-\text{Em} \) collisions.

(iii) Although \( \langle n_b \rangle \) is roughly the same for \( p-\text{Em} \), \( \alpha-\text{Em} \) and \(^{12}\text{C-Em}\) collisions, the shapes of \( n_b \) distributions are different. The \( n_b \) distribution for \(^{12}\text{C-Em}\) collisions is
Fig. 3.3 Multiplicity distributions of shower, grey, black and heavy particles in p-Em, α-Em and 12C-Em collisions at 3.0, 2.1 and 4.5 GeV/c respectively.
Table 3.3
Average multiplicities of charged particles in different ensembles of $^{12}\text{C-Em}$ (4.5 A GeV/c), $^{56}\text{Fe-Em}$ (2.5 A GeV/c) and p-Em (4.5 GeV/c) collisions

<table>
<thead>
<tr>
<th>Collisions</th>
<th>$N_h$</th>
<th>$&lt;n_b&gt;$</th>
<th>$&lt;n_g&gt;$</th>
<th>$&lt;n_s&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-Em</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{12}\text{C-Em}$</td>
<td>$N_h \geq 0$</td>
<td>3.77 ± 0.08</td>
<td>2.81 ± 0.06</td>
<td>1.63 ± 0.02</td>
</tr>
<tr>
<td>$^{56}\text{Fe-Em}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>p-Em</td>
<td>$N_h \leq 6$</td>
<td>4.45 ± 0.14</td>
<td>6.05 ± 0.18</td>
<td>7.71 ± 0.23</td>
</tr>
<tr>
<td>$^{12}\text{C-Em}$</td>
<td>CNO target</td>
<td>1.39 ± 0.04</td>
<td>1.21 ± 0.03</td>
<td>1.68 ± 0.06</td>
</tr>
<tr>
<td>$^{56}\text{Fe-Em}$</td>
<td>CNO target</td>
<td>1.84 ± 0.07</td>
<td>2.91 ± 0.13</td>
<td>8.03 ± 0.33</td>
</tr>
<tr>
<td>p-Em</td>
<td>$6 &lt; N_h \leq 15$</td>
<td>5.96 ± 0.09</td>
<td>4.40 ± 0.06</td>
<td>1.66 ± 0.06</td>
</tr>
<tr>
<td>$^{12}\text{C-Em}$</td>
<td>AgBr target</td>
<td>8.47 ± 0.37</td>
<td>10.99 ± 0.48</td>
<td>11.28 ± 0.49</td>
</tr>
<tr>
<td>$^{56}\text{Fe-Em}$</td>
<td>AgBr target</td>
<td>7.38 ± 0.18</td>
<td>14.90 ± 0.50</td>
<td>19.60 ± 0.60</td>
</tr>
<tr>
<td>p-Em</td>
<td>$N_h \geq 15$</td>
<td>11.80 ± 0.17</td>
<td>7.96 ± 0.15</td>
<td>1.29 ± 0.06</td>
</tr>
<tr>
<td>$^{12}\text{C-Em}$</td>
<td>$N_h \geq 28$</td>
<td>14.50 ± 1.20</td>
<td>21.41 ± 1.80</td>
<td>15.20 ± 1.28</td>
</tr>
<tr>
<td>$^{56}\text{Fe-Em}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
enriched by small and large values of \( n_b \).

(iv) There is a dip in the \( N_h \) distribution for \( ^{12}_C \)-Em collisions at \( N_h \approx 3-4 \).

(v) In the \( N_h \) distribution, most of the events in p-nucleus case are populated at lower values of \( N_h \) whereas the distribution in the case of nucleus-nucleus collisions have a good proportion of events at higher \( N_h \) values. The maximum \( N_h \) value observed is 59 for \( ^{12}_C \)-Em collisions in comparison to 26 and 36 for p-Em and \( \alpha \)-Em collisions respectively.

3.2.4 Dependence of Shower Particle Multiplicity on \( n_g \)

In Fig. 3.4 we plot shower multiplicity distribution in different \( n_g \) intervals to check the dependence of shower multiplicity on \( n_g \). It is clear from the figure that the peak of the distribution shifts towards higher value of \( n_s \) with the increase in \( n_g \). Similar results are also reported for proton-nucleus collisions (12,13) and nucleus-nucleus collisions at different energies (2,9).

3.2.5 Charged Particle Multiplicity Correlations

It is well known that in the study of correlations of secondary charged particles produced in collisions of high energy hadrons with nuclei, it is possible to obtain extremely useful information on the dynamics of collision and it allows us to discuss the mechanism of nucleus-nucleus collisions. According to the existing representation, the shower and grey
Fig. 7.4 Multiplicity distribution of shower particles in different n_g-intervals.
particles characterize the fast stage of the inelastic collision between two nuclei, black particles correspond to the next stage of collision when the de-excitation process occurs through the evaporation of nucleons.

Several workers (14-18) have attempted to study the multiplicity correlations over widely different incident energies and using different projectiles. An analysis of multiplicity correlations in the range 20-200 GeV/c for p-nucleus collisions was done by Azimov et al (19). It showed that the inclination coefficients \( \langle n_i \rangle \) are monotonic and can reasonably be approximated by lines with positive slopes. AALMT collaboration (13) studied the charged particle multiplicity correlations at 200 GeV/c for p-Em collisions and observed that all the inclination coefficients are monotonic with positive slopes. A comparison of the data with low energy results (20) shows full agreement with the correlations between the black and grey track multiplicities. A similar type of study has also been carried out at 24 and 400 GeV/c for p-Em collisions (21). The study reveals that the correlations between multiplicities of slow particles do not depend on the energy of the projectile. The values of inclination coefficients are positive and are in fairly good agreement with the corresponding values reported in Ref. (16). Ahrar et al (22) also studied the multiplicity correlations for p-Em collision at 400 GeV/c. These correlations may be represented satisfactorily by linear functions with positive
slopes. They compared their result with the result at 200 GeV/c (14) and found that the coefficients of inclination continued to grow with incident energy. We have also studied correlations among different multiplicity parameters with a view to ascertaining continuity from proton-nucleus to nucleus-nucleus collision mechanism. The correlations of the multiplicities $n_i = f(n_j)$ and their approximation by linear dependences $n_i = q + kn_j$ are given in Fig. 3.5 for $^{12}$C-Em collisions and in Fig. 3.6 for p-Em (8) collisions. These correlations like in hadron-nucleus collisions, can be represented by linear relations with positive slopes. A similar result was obtained by Otterlund (23) for p-nucleus collisions over a wide range of energy. It means that the correlations do not depend on the mass of the projectile and the contribution of the recoiling nucleus towards the excitation energy of the residual nucleus is approximately the same for p-nucleus and nucleus-nucleus collisions.

The least square fits of the experimental points have also been indicated in the figures. The equations representing these fits are:

For $<n_b>$, $<n_g>$ and $<N_h>$ as a function of $n_s$.

\[
<n_b> = (0.41 \pm 0.07) \, n_s + (1.41 \pm 0.17)
\]

\[
<n_g> = (0.92 \pm 0.08) \, n_s + (-1.13 \pm 0.15)
\]

\[
<N_h> = (1.69 \pm 0.11) \, n_s + (1.57 \pm 0.19)
\]
Fig. 3.5 Multiplicity correlations \( n_i = f(n_j) \) and their approximations by linear dependence \( n_i = c + kn_j \) for \(^{12}\text{C}\)-Em collisions.
Fig. 3.5 Multiplicity correlations $n_i = f(n_j)$ and their approximation by linear dependence $n_i = c + kn_j$ for p-Em collisions.
For \( <n_g>, <n_s> \) and \( <N_h> \) as a function of \( n_b \).

\[
\begin{align*}
  <n_g> &= (0.82 \pm 0.10) n_b + (3.14 \pm 0.21) \\
  <n_s> &= (0.45 \pm 0.08) n_b + (7.32 \pm 0.38) \\
  <N_h> &= (1.92 \pm 0.16) n_b + (2.69 \pm 0.23)
\end{align*}
\]

For \( <n_b>, <n_s> \) and \( <N_h> \) as a function of \( n_g \).

\[
\begin{align*}
  <n_b> &= (0.31 \pm 0.06) n_g + (3.82 \pm 0.29) \\
  <n_s> &= (0.49 \pm 0.08) n_g + (6.32 \pm 0.44) \\
  <N_h> &= (1.38 \pm 0.12) n_g + (3.45 \pm 0.36)
\end{align*}
\]

For \( <n_b>, <n_g> \) and \( <n_s> \) as a function of \( N_h \).

\[
\begin{align*}
  <n_b> &= (0.46 \pm 0.09) N_h + (0.19 \pm 0.04) \\
  <n_g> &= (0.75 \pm 0.12) N_h + (-0.95 \pm 0.06) \\
  <n_s> &= (0.51 \pm 0.07) N_h + (6.77 \pm 0.47)
\end{align*}
\]

In Table 3.4, we have given the values of the inclination coefficients for \( ^{12}\text{C}-\text{Em} \) and \( p-\text{Em} \) collisions at the same incident momentum. The following conclusion can be drawn from Figs. 3.5 and 3.6 and the data presented in Table 3.4.

(i) In the case of \( p-\text{Em} \) collisions the correlation between \( <N_h> \) and \( n_b \) is very strong, between \( <n_g> \) and \( n_b \) moderate and negative between \( <n_s> \) and \( n_b \). In the case of \( ^{12}\text{C}-\text{Em} \) collisions the correlation between \( <N_h> \) and \( n_b \) is strong and moderate between \( ( <n_s> , <n_g> ) \) and \( n_b \).
Table 3.4

Values of the inclination coefficients in the multiplicity correlations in case of $^{12}$C-Em and p-Em collisions

<table>
<thead>
<tr>
<th>Collisions</th>
<th>$&lt;n_b&gt;$</th>
<th>$&lt;n_g&gt;$</th>
<th>$&lt;n_s&gt;$</th>
<th>$&lt;N_h&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_b$</td>
<td>p-Em</td>
<td>$0.51 \pm 0.01$</td>
<td>$-0.03 \pm 0.01$</td>
<td>$1.52 \pm 0.02$</td>
</tr>
<tr>
<td></td>
<td>$^{12}$C-Em</td>
<td>$0.82 \pm 0.10$</td>
<td>$0.45 \pm 0.08$</td>
<td>$1.92 \pm 0.16$</td>
</tr>
<tr>
<td>$n_g$</td>
<td>p-Em</td>
<td>$1.09 \pm 0.02$</td>
<td></td>
<td>$-0.06 \pm 0.01$</td>
</tr>
<tr>
<td></td>
<td>$^{12}$C-Em</td>
<td>$0.31 \pm 0.06$</td>
<td></td>
<td>$0.49 \pm 0.08$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>p-Em</td>
<td>$0.01 \pm 0.04$</td>
<td>$-0.04 \pm 0.04$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^{12}$C-Em</td>
<td>$0.41 \pm 0.07$</td>
<td>$0.92 \pm 0.08$</td>
<td></td>
</tr>
<tr>
<td>$N_h$</td>
<td>p-Em</td>
<td>$0.60 \pm 0.01$</td>
<td>$0.40 \pm 0.01$</td>
<td>$-0.02 \pm 0.00$</td>
</tr>
<tr>
<td></td>
<td>$^{12}$C-Em</td>
<td>$0.46 \pm 0.09$</td>
<td>$0.75 \pm 0.12$</td>
<td>$0.51 \pm 0.07$</td>
</tr>
</tbody>
</table>
(ii) There is negative correlation between $n_g$ and $\langle n_s \rangle$ and very strong correlation between $\langle n_b \rangle$, $\langle N_h \rangle$ and $n_g$ in the case of p-Em collisions, whereas in the case of $^{12}$C-Em collisions, there is strong correlation between $\langle N_h \rangle$ and $n_g$ and moderate correlation between $\langle n_b \rangle$, $\langle n_s \rangle$ and $n_g$.

(iii) For p-Em collisions, there is practically no dependence of $\langle n_g \rangle$, $\langle n_b \rangle$ and $\langle N_h \rangle$ on $n_s$, but in the case of $^{12}$C-Em collisions this dependence is quite strong.

(iv) For p-Em collisions, there is strong correlation between $\langle n_b \rangle$ and $N_h$ and negative correlation between $\langle n_s \rangle$ and $N_h$ whereas in the case of $^{12}$C-Em collisions $\langle n_b \rangle$, $\langle n_g \rangle$ and $\langle n_s \rangle$ increase linearly with $N_h$.

(v) The correlations between multiplicities of slow particles, i.e., between black, grey and heavy tracks seem to depend on the nature of the incident particle.

(vi) The mean shower multiplicity $\langle n_s \rangle$ at fixed $n_b$, $n_g$ and $N_h$ is larger for $^{12}$C-Em collisions than p-Em collisions.

3.2.6 Multiplicity Scaling of Shower Particles

The probability distribution for the production of $n$ charged particles in hadron-hadron collisions is observed to exhibit a universal behaviour, hence one may express

$$P(n) = \langle n \rangle^{-1} \psi \left( \frac{n}{\langle n \rangle} \right),$$

(3.2)

where $P(n)$ is the probability of producing $n$ charged particles, $\langle n \rangle$ represents the average number of charged particles and $\psi$ is some function of variable $Z = \left( \frac{n}{\langle n \rangle} \right)$. This behaviour of multiplicity distribution as a function of the variable $Z$ is referred
to as KNO scaling (24). The probability of emission of \( n \) charged particles in p-p collisions is related to the scaling function \( \psi \) as (24)

\[
\mathcal{P}(n) = \langle n \rangle^{-1} \psi(Z) = \frac{\sigma_n}{\sigma_{\text{inel}}}, \tag{3.3}
\]

independent of the energy of the incoming hadron and the mass of the target nucleus, where

\( \sigma_n = \) particle cross-section for producing \( n \) charged particles

and

\( \sigma_{\text{inel}} = \) total inelastic cross-section.

Slattery (25) has been able to express the scaling function \( \psi(Z) \) for p-p collisions in the energy range \((50-303) \) GeV as

\[
\psi(Z) = (3.79Z + 33.7Z^3 - 6.64Z^5 + 0.332Z^7) \exp(-3.04Z), \tag{3.4}
\]

Slattery (25) has further shown that this function fits the multiplicity data of p-p collisions quite well in the energy range \((50-303) \) GeV but it poorly fits the data in the lower energy range \((19-38.5) \) GeV. Martin et al (26) have, however, observed that the charged shower particle multiplicity for p-Em collisions obeyed a KNO type scaling instead of the exact KNO scaling. They have modified the functional form of \( \psi(Z) \) as

\[
\psi_m(Z) = (6.84Z + 26.6Z^3 - 2.12Z^5 + 0.164Z^7) \exp(-3.28Z), \tag{3.5}
\]

Here \( \psi_m(Z) \) stands for the modified scaling function obtained by Martin et al (26).

Olesen (27) has observed that the KNO scaling function is
not well satisfied at very low energies, at least in the case of p-p collisions. It has, therefore, been realized by several workers (28,29) that the data on multiplicity distribution of charged shower particles should be analyzed in terms of actual number of created particles instead of all the final state particles in both hadron-hadron and hadron-nucleus collisions.

In recent years many attempts (30,31) have been made to study the multiplicity distribution of shower particles produced in nucleus-nucleus collisions. Jain et al (30) were the first to study the existence of KNO type scaling in nucleus-nucleus collisions. These results might be of extreme interest to both experimentalist and theorist as the scaling may be a reflection of some unexplored or unknown phenomenon that occurs exclusively in relativistic nucleus-nucleus collisions.

The validity of KNO type scaling at different beam energies has been debated by many workers with diverse opinions (32). In order to accommodate the data at low energies, a simple empirical modification of Eq. 3.2 was proposed to extend this type of scaling (KNO) (28). Explicitly, the new scaling law is of the form

\[ P(n) = \left( \frac{1}{\langle n \rangle - \alpha} \right) \psi(Z'), \quad (3.6) \]

where \( Z' = (n-\alpha)/\langle n \rangle - \alpha \) and \( \alpha \) is a constant and is independent of energy but may depend upon the type of collision. The KNO scaling may be considered a consequence of certain properties
of moments and correlations when the particles are produced in clusters.

In order to see how far we could extend the modified KNO scaling in the lower energy range, we have studied here the scaling behaviour of shower particles in $^{12}\text{C-Em}$ and $^{12}\text{C-AgBr}$ collisions at 4.5 A GeV/c. We excluded singly charge $Z = 1$ fragments from the multiplicity distributions. The average multiplicities of shower particles $<n_s>$ for $^{12}\text{C-Em}$ and $^{12}\text{C-AgBr}$ collisions are $7.71 \pm 0.23$ and $11.28 \pm 0.49$ respectively. In Fig. 3.7, we plot $\psi(Z')$ versus $Z' = (n-\alpha)/<n>-\alpha$ for $^{12}\text{C-Em}$ and $\psi(Z)$ versus $Z = n/<n>$ for $^{12}\text{C-AgBr}$ collisions. We fitted distributions using a simple parameterization for $\psi(Z')$ and varied value of $\alpha$ in order to obtain the minimum overall $\chi^2$ per degree of freedom and found $\alpha = 0.6$. The experimental points lie on the universal curve which can be fitted with a KNO type scaling function.

For $^{12}\text{C-Em}$ collisions the functional form appears as

$$\psi(Z') = 1.0 \ (Z' + 0.46) \ \exp (-1.11Z' - 0.09Z'^2),$$

with $\chi^2/D.O.F = 0.89$

and for $^{12}\text{C-AgBr}$ collisions as

$$\psi(Z) = (1.59Z + 7.52Z^3 - 3.06Z^5 + 0.34Z^7) \ \exp (-2.12Z),$$

with $\chi^2/D.O.F = 0.37$

Equations 3.7 and 3.8 give the best fit of the experimental data. We may thus conclude that our data are consistent with
Fig. 3.7 Plot between $\psi(z')$ versus $z'$ for $^{12}\text{C}-\text{Em}$ and $^{12}\text{C}-\text{AgBr}$ collisions at 4.5 $\text{A GeV/c}$.  

$z = \frac{n}{\langle n \rangle}$  

$z' = \frac{(n-0.6)}{\langle n \rangle - 0.6}$
the scaling hypothesis.

We also studied the scaling behaviour of shower particles emitted from $\alpha$-Em(11) and $^{14}$N-Em(2) collisions at 2.1 A GeV. In Fig. 3.8 we plot $\Psi(Z')$ vs $Z'$ for $\alpha$-Em and $^{14}$N-Em collisions. The experimental points lie on the universal curve which can be fitted with a KNO type scaling function.

For $\alpha$-Em collisions at 2.1 A GeV/c, the functional form is

$$\Psi(Z') = 1.22 (Z' + 0.24) \exp (-1.37Z' - 0.01Z'^2), \quad (3.9)$$

with $\chi^2$/D.O.F = 2.04

and for $^{14}$N-Em collisions at 2.1 A GeV,

$$\Psi(Z') = 0.93 (Z' + 0.45) \exp (-0.96Z' - 0.16Z'^2), \quad (3.10)$$

with $\chi^2$/D.O.F = 0.67

Hence, these observations of scaling in the multiplicity distributions of shower particles indicate that the particle production mechanism does not depend upon the mass and energy of the projectile.

The KNO scaling described by Eq. 3.2 can also be expressed in terms of moments. The normalized moments of relativistic charged secondaries are defined as

$$C_k = \frac{\langle n_s^k \rangle}{\langle n_s \rangle^k} = \text{Constant}, \quad (3.11)$$

where $k$ can have value 1, 2, 3, 4, etc.

leads to a linear relation between the dispersion and the mean multiplicity $\langle n_s \rangle$. The dispersion of the multiplicity
Fig. 3.8 Plot between $\Psi(Z')$ versus $Z'$ for $c-\text{Em}$ and $^{14}_N-\text{Em}$ collisions at 2.1 A GeV/c.
distribution is defined as

\[
D(n_s) = \left[ \langle n_s^2 \rangle - \langle n_s \rangle^2 \right]^{1/2},
\]

(3.12)

Table 3.5 shows the values of \( \langle n_s \rangle \), \( D \), \( \langle n_s \rangle / D \) and \( C_2 \) and \( C_3 \) for \( ^{12}\text{C} - \text{Em} \) collisions. For the sake of comparison, we also show the values of \( \langle n_s \rangle \), \( D \), \( \langle n_s \rangle / D \) and \( C_2 \) and \( C_3 \) for \( \text{A-Em} \), \( \text{p-Em} \) and \( \overline{\text{p}} - \text{Em} \) collisions at different energies. From the table, it may be noted that the values of the ratio \( \langle n_s \rangle / D \) for hadron-nucleus and nucleus-nucleus collisions are not much different, indicating that the production mechanism of shower particles may be similar for both types of collisions. In the case of nucleus-nucleus collisions, it can also be noted that the dispersion increases with the mass of the projectile.

Fig. 3.9 shows a plot of \( D \) as a function of \( \langle n_s \rangle \) for \( ^{12}\text{C} - \text{Em} \) collisions observed in the present experiment and other data points of \( \text{A-Em} \), \( \text{p-Em} \) and \( \overline{\text{p}} - \text{Em} \) collisions at different energies have been taken from reference (2,8,11,14,22,30,34-36). We see that the dispersion linearly increases with increasing \( \langle n_s \rangle \) and the dispersion of nucleus-nucleus collisions increases with \( \langle n_s \rangle \) faster than hadron-nucleus collisions. An equation for the fitted line is given by \( D = (0.52 \pm 0.05) \, n_s + (0.48 \pm 0.03) \). The implication of this result is far reaching in the realm of heavy ion collisions at relativistic energies.

From Table 3.5, we can say that the values of the moments are constant within their statistical limits. However, the value
Fig. 3.9 Plot D versus $\langle n_s \rangle$ for the present work and other data points of $\pi-E_m$, $p-E_m$ and $\bar{p}-E_m$ collisions at different energies have been taken from the references (2, 8, 11, 14, 22, 30, 34-36). Equation for the least square fit for data points is given by $D = (C.52 \pm C.05) n_s + (C.49 \pm C.03)$. 
of $C_k$ increases with the increase of the value of $k$. The values of these moments for $^{12}$C-\pi collisions agree well with those of \pi-p and \nu-p collisions and with the asymptotic values of hadron-hadron collisions (37,38). Therefore, we may say that the parameters of the multiplicity distributions are close to each other in hadron-nucleus and nucleus-nucleus collisions which again shows that the production mechanism of shower particles is identical in nature in both types of collisions.

3.2.7 Dependence of Shower Particle Multiplicity on Projectile Mass

We shall now investigate the influence of projectile mass on the mean multiplicity of the shower particles. Table 3.5 presents the average multiplicities of different particles for different projectiles. It is clear from the table that $\langle n_s \rangle$ increases with the mass of the projectile nucleus. Figure 3.10 shows the dependence of $\langle n_s \rangle$ on the mass of the projectile. The $\langle n_s \rangle$ dependence on $A$ can be approximated by the power function $\langle n_s \rangle = KA^\alpha$, where $K$ and $\alpha$ are constants. The best fit values of $K$ and $\alpha$ are $(1.54 \pm 0.09), (0.60 \pm 0.30)$ respectively.

3.2.8 Dependence of Multiplicity on the Number of Interacting Projectile Nucleons

The number of projectile nucleons interacting with the target ($m$) is one of the basic parameters of the so called superposition model, where a nucleus-nucleus collision is described as a superposition of nucleon-nucleus collisions (39,40)
Fig. 3.10 The average multiplicity of shower particles as a function of the mass of the projectile, \( \tau \). The solid line shows the relation \( \langle n_\tau \rangle = \text{Const.} \ A^\alpha \). (See Text).
<table>
<thead>
<tr>
<th>Type of Collisions</th>
<th>Momentum A GeV/c</th>
<th>(&lt;n_s&gt;)</th>
<th>D</th>
<th>(&lt;n_s&gt;/D)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)-Em</td>
<td>2.1</td>
<td>3.06 ± 0.28</td>
<td>2.49 ± 0.04</td>
<td>1.23 ± 0.11</td>
<td>1.44 ± 0.04</td>
<td>2.55 ± 0.09</td>
<td>33</td>
</tr>
<tr>
<td>(^{12})C-(\alpha)</td>
<td>4.5</td>
<td>7.71 ± 0.23</td>
<td>5.66 ± 0.11</td>
<td>1.46 ± 0.05</td>
<td>1.58 ± 0.07</td>
<td>2.67 ± 0.13</td>
<td>Present work</td>
</tr>
<tr>
<td>(^{12})C-AgBr</td>
<td>4.5</td>
<td>11.28 ± 0.49</td>
<td>6.09 ± 0.12</td>
<td>1.85 ± 0.09</td>
<td>1.29 ± 0.06</td>
<td>2.48 ± 0.08</td>
<td>2</td>
</tr>
<tr>
<td>(^{14})N-Em</td>
<td>2.1</td>
<td>8.85 ± 0.28</td>
<td>5.59 ± 0.17</td>
<td>1.58 ± 0.07</td>
<td>1.40 ± 0.05</td>
<td>2.58 ± 0.11</td>
<td>30</td>
</tr>
<tr>
<td>(^{40})Ar-(\alpha)</td>
<td>1.85</td>
<td>15.38 ± 0.16</td>
<td>9.0 ± 0.21</td>
<td>1.71 ± 0.04</td>
<td>1.30 ± 0.02</td>
<td>1.99 ± 0.06</td>
<td>9</td>
</tr>
<tr>
<td>(^{56})Fe-(\alpha)</td>
<td>2.5</td>
<td>13.30 ± 0.40</td>
<td>12.64 ± 0.29</td>
<td>1.05 ± 0.12</td>
<td>-</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>p-Em</td>
<td>4.5</td>
<td>1.63 ± 0.02</td>
<td>1.08 ± 0.02</td>
<td>1.51 ± 0.03</td>
<td>1.44 ± 0.02</td>
<td>2.55 ± 0.04</td>
<td>19</td>
</tr>
<tr>
<td>p-(\alpha)</td>
<td>50</td>
<td>8.79 ± 0.10</td>
<td>5.18 ± 0.10</td>
<td>1.68 ± 0.04</td>
<td>-</td>
<td>-</td>
<td>22</td>
</tr>
<tr>
<td>p-(\alpha)</td>
<td>400</td>
<td>16.86 ± 0.51</td>
<td>9.51 ± 0.38</td>
<td>1.77 ± 0.06</td>
<td>-</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>p-(\alpha)</td>
<td>≥ 1000</td>
<td>15.18 ± 0.15</td>
<td>7.60 ± 0.20</td>
<td>1.99 ± 0.06</td>
<td>1.27 ± 0.02</td>
<td>1.93 ± 0.09</td>
<td>34</td>
</tr>
<tr>
<td>(\pi)-Em</td>
<td>50</td>
<td>8.27 ± 0.12</td>
<td>4.60 ± 0.07</td>
<td>1.79 ± 0.04</td>
<td>1.31 ± 0.04</td>
<td>2.14 ± 0.06</td>
<td>14</td>
</tr>
<tr>
<td>(\pi)-Em</td>
<td>200</td>
<td>11.94 ± 0.23</td>
<td>6.98 ± 0.11</td>
<td>1.72 ± 0.08</td>
<td>1.33 ± 0.03</td>
<td>2.22 ± 0.09</td>
<td>35</td>
</tr>
<tr>
<td>(\pi)-Em</td>
<td>340</td>
<td>14.22 ± 0.12</td>
<td>7.50 ± 0.13</td>
<td>1.89 ± 0.09</td>
<td>1.47 ± 0.05</td>
<td>2.23 ± 0.09</td>
<td></td>
</tr>
</tbody>
</table>
Therefore, it is interesting to study the dependence of average multiplicities of different particles on the number of nucleons \( m \) of the projectile that have interacted with the target. This number can roughly be estimated using the relation \( m = 12 - 2Q \), where \( Q = \sum N_i Z_i \) is the total charge of the projectile fragments and \( N_i \) is the number of fragments with charge \( Z_i \). The minimal value of \( Q \) is \( Q^\text{min} = n_{Z=1} + 2n_{Z=2} + 3n_{Z=3} \), where \( n_Z \) is the number of fragments with fixed \( Z \). Therefore, we will use the quantity

\[
m = \begin{cases} 
1 & \text{if } Q \geq 6 \\
12 - 2Q & \text{if } Q < 6 
\end{cases}
\]

For our \(^{12}\text{C-Em}\) collisions, the mean value of \( m \) was found to be \( 7.01 \pm 0.27 \). Figure 3.11 shows the dependence of the average multiplicities of shower, grey and black particles on \( m \). As can be seen from the figure, \( \langle n_s \rangle, \langle n_g \rangle \) and \( \langle n_b \rangle \) increase monotonically with \( m \) and for \( \langle n_s \rangle \) this increase is approximately linear. The mean multiplicity of shower particles per interacting nucleon \( \langle n_s \rangle/m \) is approximately constant and is equal to \( 1.23 \pm 0.02 \). It compares well with \( 1.63 \pm 0.02 \) observed in \( p\text{-Em} \) collisions at \( 4.5 \text{ GeV/c} \) (41) and 1.1 in \(^{14}\text{N-Em}\) collisions at \( 2.1 \text{ A GeV} \) (2).

3.2.9 Angular Distribution of Shower Particles

The angular distribution of shower particles produced in \(^{12}\text{C-Em}\) collisions at \( 4.5 \text{ A GeV/c} \) is shown in Fig. 3.12a in comparison with those for \( p\text{-Em} \) at \( 3.0 \text{ GeV/c} \) (10) and \( \alpha\text{-Em} \) at
Fig. 3.11 The dependence of $\langle n_s \rangle$, $\langle n_s \rangle/m$, $\langle n_g \rangle$ and $\langle n_b \rangle$ on the number of nucleons of the projectile interacting with the target.
2.1 A GeV/c (11). These distributions are similar except at small angles where a contribution of singly charged fragments enhances the number of shower particles in the case of nucleus-nucleus collisions. From the similarity of these distributions we can conclude that the shower production mechanism is of similar nature and is independent of the nature, size and energy of the projectile.

3.2.10 Angular Distribution of Target Fragments

The angular distributions of grey and black particles are shown in Fig. 3.12(b,c). For the sake of comparison, the angular distributions of grey and black particles for p-Em collisions at 3.0 GeV/c and α-Em collisions at 2.1 A GeV/c are also shown in the same figure. It can be seen from the figure that there is no dependence of these distributions on the mass of the projectile. This indicates that the production mechanism of heavy particles is probably the same in p-nucleus and nucleus-nucleus collisions. It can also be seen that these distributions do not exhibit any peaks that could be attributed to the shock-wave phenomenon (42-45).

3.3 Conclusions

We have studied a sample of 13CO $^{12}$C-Em collisions at 4.5 A GeV/c. The results have been compared with relevant data from collisions of other projectiles with emulsion. An attempt has been made to find systematics in the results. The following conclusions are drawn from the results presented in this Chapter.
Fig. 3.12 Angular distributions of shower, grey and black particles in $\kappa$-$\Xi m$, $\alpha$-$\Xi m$ and $^{12}$C-$\Xi m$ collisions at 3.0, 2.1 and 4.5 $\times$ GeV/c respectively.
(i) The mean multiplicity of black particle \( <n_b> \) does not depend on the mass of the projectile. However, the mean multiplicity of grey particles \( <n_g> \) depends on the mass of the projectile.

(ii) It is found that the average multiplicities of shower and grey particles increase with the mass of the projectile and the dependence can be described by the relation of type \( <n> = \text{Const.} \ A^{\alpha} \).

(iii) The peak of the \( n_s \) distribution shifts towards higher values of \( n_s \) with increase in \( n_g \).

(iv) The charged particle multiplicity correlations in \( ^{12}\text{C}-\text{Em} \) collisions are like those in hadron-nucleus collisions.

(v) The correlations between multiplicities of slow particles, i.e., between black, grey and heavy tracks seem to depend on the nature of the incident particle.

(vi) In the case of \( ^{12}\text{C}-\text{Em} \) collisions, the shower particle multiplicity distribution obeys a KNO type scaling law.

(vii) The average number of interacting nucleons of projectile is found to be \( (7.01 \pm 0.27) \) in the case of \( ^{12}\text{C}-\text{Em} \) collisions and the mean multiplicity of shower particles per interacting nucleon is found to be approximately the same as the multiplicity of shower particles in p-Em collisions at the same energy.

(viii) The angular distributions of shower, grey and black particles do not depend on the mass of the projectile.

(ix) The angular distributions of grey and black particles show no significant peaks which could be attributed to the shock-wave phenomenon.
REFERENCES

CHAPTER-IV

CENTRAL $^{12}$C-Em COLLISIONS AT 4.5 A GeV/c

4.1 Introduction

A lot of effort has gone into the study of peripheral collisions of relativistic nuclei and many interesting and important results have been obtained. However, the central collisions of relativistic nuclei, in which almost the whole projectile takes part in the collision, have not received the desired attention. This is partly due to the fact that the probability of occurrence of these collisions is small ($\sim 1\%$), leading to low statistics and partly due to the fact that a lot of complex phenomena might occur during these collisions, making the analysis of these events very difficult. Nevertheless, the study of central collisions is very important because during these collisions, the nuclear matter might be compressed to several times its normal density and consequently several interesting phenomena, e.g., shock-wave, production of quark-gluon plasma etc. might occur. Therefore, a study of central collision of relativistic nuclei is expected to shed some light on these exotic phenomena. Moreover, some characteristics of central collisions are more critical to the choice of the collision model. Therefore, data on central collisions could also be used in refining the existing models of nucleus-nucleus collisions.
In this Chapter, we study the central $^{12}\text{C}$-Em collisions at 4.5 A GeV/c using a sample of 1300 events. Multiplicity and angular distributions of charged secondaries have been studied in detail and compared to proton-nucleus and nucleus-nucleus data. The correlations among various multiplicity parameters have been studied. Angular distributions of grey and black particles have also been studied with a view to finding if there are any peaks in the distributions which could be attributed to the shock-wave phenomenon.

4.2 Experimental Results

4.2.1 Probability of Central Collisions

There are no standard criteria for selecting central collisions. Different workers have used different criteria for defining central collisions. Heckman et al (1) have studied the central collisions of $^4\text{He}$, $^{12}\text{C}$, $^{14}\text{N}$ and $^{16}\text{O}$ nuclei with emulsion at 2.1 A GeV. They defined central collisions as the events which have no $Z \geq 2$ projectile fragments emitted within $5^\circ$ of the beam direction. For p-nucleus collisions, Barashenkov et al (2) used the multiplicity of heavy particles ($N_h \geq 28$) for selecting central collisions. The same criterion has been used by many workers (3-8) to select central collisions in the case of nucleus-nucleus collisions also. However, in the case of nucleus-nucleus collisions there is an additional group of secondary particles, the projectile fragments of charge
which are emitted within a small angle with respect to the beam direction. Thus the criterion for central collisions (\(N_h \geq 28\)) which was introduced for p-nucleus collisions should be modified in the case of nucleus-nucleus collisions. We therefore defined central collisions as the events with \(N_h \geq 28\) and having no observable projectile fragments, even singly charged one, emitted within 3° of the beam direction. Out of 1300, only 152 events satisfied this criteria. This comes to 11.7% of all events. These are complete central collisions except in a few cases some projectile neutrons may have passed through the target nucleus without collision while all protons collided. However, these are most central events studied so far. Table 4.1 presents the probability of central collisions for different projectiles. It can be seen from the table that the probability increases with the mass of the projectile.

4.2.2 Multiplicities of Secondary Particles in Central \(^{12}\text{C-Em Collisions}\)

Table 4.2 presents the average multiplicities of different types of secondary particles produced in central collisions for different projectiles. It can be seen from the table that the average multiplicity of grey particles, \(\langle n_g \rangle\), increases while the average multiplicity of black particles, \(\langle n_b \rangle\), decreases with the increases in projectile mass. This result can be explained in terms of the fireball model (9). According to the model, the grey particles come from the participant volume.
and the number of participant nucleons increases as the volume of the cylinder cut in the target by the projectile increases. This volume increases with the increase in projectile mass and consequently the average number of grey particles, \( <n_g> \), increases. Since the size of the target nucleus is limited, the number of black particles decreases when the number of grey particles increases.

In Table 4.2 we have also given the value of \( E \), the energy available in the centre of mass system for the production of secondary particles. It was estimated as follows. According to the fireball model (9), the total energy in the centre of mass system is given by

\[
E_{\text{c.m}} = \left[ (N_p + N_T)^2 \ m'^2 + 2 N_p N_T \ T \ m' \right]^{1/2}, \tag{4.1}
\]

where \( N_p \) and \( N_T \) are the numbers of participant nucleons of the projectile and the target respectively, \( T \) is the kinetic energy per nucleon of the projectile and \( m' \) is the mass of the bound nucleon. For central \( ^{12}\text{C}-\text{Em} \) collisions, \( N_p = 12 \), and \( N_T \) is the number of nucleons in the cylinder cut in the target by the projectile and is given by \( N_T = 1.5 \ \frac{A_p^{2/3}}{A_T^{1/3}} \), where \( A_p \) and \( A_T \) are the mass numbers of the projectile and target respectively. The energy available in the centre of mass system for the production of secondary particles is then given by

\[
E = E_{\text{c.m}} - (N_p + N_T) \ m', \tag{4.2}
\]
where \( m \) is the nucleon mass.

As can be seen from Table 4.2, there is a strong correlation between the energy available in the centre of mass system, \( E \) and the average multiplicity of shower particles, \( \langle n_s \rangle \). In Fig. 4.1, \( \langle n_s \rangle \) is plotted against \( E \) for different projectiles. The data were fitted to the relation \( \langle n_s \rangle = a + b \ln E \).

\[
a = -(10.0 \pm 2.2) \quad \text{and} \quad b = (8.1 \pm 0.9)
\]

give best fit to the data. This is the universal relation for central collisions of hadrons and nuclei.

Figures 4.2 and 4.3 show the multiplicity distributions of shower, grey, black and heavy particles produced in central \( ^{12}\text{C-Em} \) collisions at 4.5\( \text{A GeV/c} \). In Fig. 4.2, the solid curve 2 is the universal multiplicity distribution of shower particles for \( p-p \) collisions, obtained according to the KNO scaling (10) and rescaled to our \( n_s \)-distribution. It is narrower than the corresponding distribution for \( p-p \) collisions. Curve 1 is a Poisson distribution, \( P(n_s) = (\langle n_s \rangle^n/n!) \exp (-\langle n_s \rangle) \), where \( \langle n_s \rangle = (15.2 \pm 1.28) \) and \( n = 0, 1, 2, \ldots \). Within statistical errors, our \( n_s \)-distribution agrees with the Poisson distribution. This result is the agreement with the prediction of Gyulassy and Kauffmann (11) who have shown that for a thermodynamical fireball and a wide range of dynamical models, the Poisson multiplicity distribution is expected for fixed impact parameter nucleus-nucleus collisions.
Fig. 4.1 Average multiplicity of shower particle, \( \langle n_s \rangle \) versus the energy available in the centre of mass system, \( E \), for the production of secondary particles. The solid line represents the equation

\[
\langle n_s \rangle = -(10.0 \pm 2.2) + (8.1 \pm 0.9) \ln E.
\]
Fig. 4.2 Multiplicity distribution of shower particles in $^{12}\text{C}-\text{Em}$ (central) collisions. Curve 2 is a universal multiplicities distributions for pp collisions. Curve 1 is a Poisson distribution for $\langle n_s \rangle = 15.2 \pm 1.2$ (See the Text).
Fig. 4.3 Multiplicity distribution of grey, black and heavy particles in $^{12}_C$-$^{Em}$ (central) collisions at 4.5 A GeV/c.
Table 4.1
Probability of central collisions for p-Em and A-Em collisions

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Projectile momentum A GeV/c</th>
<th>Probability (%</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>7.1</td>
<td>2.1 ± 0.4</td>
<td>16</td>
</tr>
<tr>
<td>Deuteron</td>
<td>4.5</td>
<td>2.6 ± 0.5</td>
<td>17</td>
</tr>
<tr>
<td>α-Particle</td>
<td>4.5</td>
<td>6.8 ± 0.9</td>
<td>17</td>
</tr>
<tr>
<td>Carbon</td>
<td>4.5</td>
<td>11.7 ± 1.0</td>
<td>Present work</td>
</tr>
<tr>
<td>Z &gt; 6</td>
<td></td>
<td>17.9 ± 3.1</td>
<td></td>
</tr>
<tr>
<td>3 ≤ Z ≤ 5</td>
<td></td>
<td>9.3 ± 2.9</td>
<td></td>
</tr>
<tr>
<td>6 ≤ Z ≤ 9</td>
<td></td>
<td>7.8 ± 1.5</td>
<td>18</td>
</tr>
<tr>
<td>10 ≤ Z ≤ 15</td>
<td></td>
<td>11.7 ± 3.7</td>
<td></td>
</tr>
<tr>
<td>16 ≤ Z ≤ 26</td>
<td></td>
<td>13.0 ± 6.3</td>
<td></td>
</tr>
</tbody>
</table>

* Cosmic ray data

Table 4.2
Average multiplicities of different secondary particles emitted in central p-Em and nucleus-nucleus collisions

<table>
<thead>
<tr>
<th>Type of Collision</th>
<th>Projectile momentum A GeV/c</th>
<th>&lt;n_s&gt;</th>
<th>E</th>
<th>&lt;n_g&gt;</th>
<th>&lt;n_p&gt;</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>p+Em</td>
<td>7.1</td>
<td>3.4 ± 0.4</td>
<td>4.1C</td>
<td>11.3 ± 0.7</td>
<td>18.9 ± 0.8</td>
<td>16</td>
</tr>
<tr>
<td>d+Em</td>
<td>4.5</td>
<td>2.9 ± 0.2</td>
<td>5.1C</td>
<td>16.4 ± 0.4</td>
<td>16.2 ± 0.2</td>
<td>17</td>
</tr>
<tr>
<td>α+Em</td>
<td>4.5</td>
<td>6.6 ± 0.2</td>
<td>9.42</td>
<td>19.1 ± 0.4</td>
<td>14.4 ± 0.3</td>
<td>17</td>
</tr>
<tr>
<td>C+Em</td>
<td>4.5</td>
<td>15.2 ± 1.28</td>
<td>24.9C</td>
<td>21.4 ± 1.8</td>
<td>14.5 ± 1.2</td>
<td>Present work</td>
</tr>
<tr>
<td>C+Em</td>
<td>4.5</td>
<td>25.1 ± 1.1</td>
<td>32.98</td>
<td>26.2 ± 1.1</td>
<td>11.0 ± 0.5</td>
<td>20</td>
</tr>
<tr>
<td>Mg+Em</td>
<td>4.5</td>
<td>28.9 ± 0.9</td>
<td>47.10</td>
<td>23.2 ± 0.7</td>
<td>14.8 ± 0.4</td>
<td>8</td>
</tr>
<tr>
<td>d+T</td>
<td>4.5</td>
<td>2.8 ± 0.2</td>
<td>5.30</td>
<td>16.4 ± 0.4</td>
<td>16.2 ± 0.3</td>
<td>19</td>
</tr>
<tr>
<td>α+T</td>
<td>4.5</td>
<td>6.5 ± 0.2</td>
<td>9.60</td>
<td>19.1 ± 0.4</td>
<td>14.4 ± 0.3</td>
<td>19</td>
</tr>
<tr>
<td>C+T</td>
<td>4.5</td>
<td>18.9 ± 0.7</td>
<td>26.40</td>
<td>22.5 ± 0.8</td>
<td>11.9 ± 0.5</td>
<td>19</td>
</tr>
</tbody>
</table>
4.2.3 Charged Particle Multiplicity Correlations in Central $^{12}_{C}$-Em Collisions

The experimentally obtained multiplicity correlations of the type $<n_i(n_j)>$ ($i,j = s,g,b$) for $^{12}_{C}$-Em collisions are shown in Fig. 4.4.

From the results presented in Fig. 4.4, we conclude that:

(i) Shower and grey particles have similar multiplicity dependence on $n_b$ (Fig. 4.4a).

(ii) $<n_s>$ increases almost linearly with $n_g$. Thus $n_g$ may be considered to be a more reliable indicator for intranuclear collisions in nucleus-nucleus collisions as in hadron-nucleus collision (12) (Fig. 4.4b).

(iii) The number of black particles decreases and the number of grey particles increases with increasing shower particle multiplicities (Fig. 4.4c).

(iv) All the correlations of secondary particles have negative coefficients of inclination except $<n_s> = f(n_g)$. Therefore, these central $^{12}_{C}$-Em correlations are different from all $^{12}_{C}$-Em correlations, where all the inclination coefficients are positive.

4.2.4 Angular Distributions of Secondary Particles in Central $^{12}_{C}$-Em Collisions

Figure 4.5a shows the angular distribution of shower particles for central and all $^{12}_{C}$-Em collisions. On the same figure, the angular distribution of shower particles for p-Em collisions at 3.0 GeV/c (13) is also plotted. It can be seen from the figure that the behaviour of the distribution is the same in all cases and therefore we may conclude that shower
Fig. 4.4 Multiplicity correlations $\langle n_1(n_j) \rangle$ in $^{12}$C-im (central) collisions at $4.5$ $\text{A GeV/c}$.
production mechanism is independent of the impact parameter as well as the size of the projectile.

Figure 4.5b shows the angular distribution of grey particles for central and all \(^{12}\text{C}-\text{Em}\) collisions. For the sake of comparison, the angular distribution of grey particles for \(p-\text{Em}\) collisions at 3.0 GeV/c (13) is also shown in the same figure. It is clear from the figure that the angular distribution of grey particles is independent of the projectile and target mass. A similar result has been observed by other workers also (14). The values of forward to backward ratio \(F/B\) of grey particles for central and all collisions are 2.93 ± 0.11 and 3.26 ± 0.09 respectively. The forward to backward ratio \(F/B\) is defined as the number of such tracks emitted at angles <90° to those emitted at angle >90°.

In Fig. 4.5c the angular distributions of black particles for our central and all \(^{12}\text{C}-\text{Em}\) collisions and \(p-\text{Em}\) collisions at 3.0 GeV/c (13) are plotted. All these distributions are consistent with each other. The values of forward to backward ratio \(F/B\) for central and all collisions are 1.41 ± 0.06, 1.45 ± 0.04 respectively. These values indicate an approximate isotropy of black particles' emission. For \(\alpha-\text{Em}\) collisions at 4.5 A GeV/c (4), the value of \(F/B = 1.27 ± 0.30\) whereas it is equal to 1.32 ± 0.05 for \(p-\text{Em}\) collisions at 3.0 GeV/c (13). These values indicate that the angular distribution of black particles does not depend on the projectile and target mass. A similar result has been obtained by Berkeley group at different energies (1).
Fig. 4.3 Angular distributions of shower, grey and black particles in p-em, $^{12}$C-em (all) and $^{12}$C-em (central) collisions.
4.3 Conclusions

From the analysis of central $^{12}$C-Em collisions, we draw the following conclusions.

(i) The probability of central collisions increases with the mass of the projectile. This result can be explained by the fact that at high energies the inelastic cross-section is independent of energy and it increases with the mass of the projectile.

(ii) For central $^{12}$C-Em collisions, the average multiplicity of grey particles, $<n_g>$, increases while that of black particles, $<n_b>$, decreases with the mass of the projectile. This can be explained on the basis of the fireball model.

(iii) There is strong correlation between the average shower particle multiplicity, $<n_s>$, and the energy available in the centre of mass system, $E$. The relation $<n_s> = - (10.0 \pm 2.2) + (8.1 \pm 0.9) \ln E$ gives best fit to the data.

(iv) The charged particles' multiplicity correlations in central $^{12}$C-Em collisions are different from those in all $^{12}$C-Em collisions.

(v) The angular distributions of shower, grey and black particles do not depend on the mass of the projectile and the target.

(vi) The angular distributions of grey and black particles do not show any significant peaks which could be attributed to the shock-wave phenomenon.
REFERENCES

CHAPTER-V

RAPIDITY GAP DISTRIBUTIONS AT 4.5 A GeV/c

5.1 Introduction

It is now well established that multiparticle production in nucleon-nucleon and nucleon-nucleus collisions can be understood in terms of the cluster model. According to the model, a hadron system (cluster) is produced when a nucleon collides with another nucleon. This cluster then achieves the asymptotic state of free secondary particles. Particles from the decay of different clusters overlap with each other on the rapidity scale. This gives rise to short-range correlations among the secondary particles. The short-range correlations in π-nucleon and π-nucleus collisions at accelerator energies and in nucleon-nucleon and nucleon-nucleus collisions at accelerator, ISR, collider and cosmic ray energies have been extensively studied (1-17). However, only a few attempts have been made to study correlations in nucleus-nucleus collisions (18-20). Recently Kapoor et al (20) found evidence of strong correlations between particles produced in α-nucleus collisions at cosmic ray energies. This indicates that the production of particles in nucleus-nucleus collisions also takes place via cluster formation. However, in the case of nucleus-nucleus collisions, unlike nucleon-nucleon or nucleon-nucleus collisions, there are several nucleons in the projectile. A question that naturally arises is: Whether the nucleons in the projectile and
the target act independently of each other or they interact collectively? A study of correlations in nucleus-nucleus collisions would provide an answer to this question.

The study of correlations among secondary particles produced in nucleus-nucleus collisions assumes importance in view of the fact that conditions of high density and temperature could be achieved in nucleus-nucleus collisions (21-26). Under such conditions the nuclear matter may undergo a phase transition to quark-gluon plasma (27). A question that arises is: How can one prove experimentally the existence of quark-gluon plasma? Or what are the finger-prints of such a plasma? Different possibilities have been suggested by various workers. Gyulassy (28) has shown that during a central nucleus-nucleus collision, a quark-gluon plasma might be produced in the cylinder cut by the projectile in the target. When the plasma cools down it condensates into hadronic droplets (clusters). Each of these hadronic drops then decays into a large number of hadrons. Therefore, if a quark-gluon plasma is produced in nucleus-nucleus collisions, one should observe higher order correlations among the particles produced with large correlation density than one sees in case of nucleon-nucleus collisions at (50-400) GeV/c. In order to check this point and to address to the question raised earlier, we study in this chapter correlations among the secondary particles produced in $^{12}$C—Em and central $^{12}$C—Em collisions at 4.5 A GeV/c.
One can study the existence of clustering in high energy multiparticle production through rapidity gap distributions. The rapidity $Y$ of a particle is defined as

$$Y = \frac{1}{2} \ln \left( \frac{E}{P_T} + \frac{P_T}{P_T} \right), \quad (5.1)$$

where $E$ and $P_T$ are the energy and longitudinal momentum of the secondary particle respectively. At very high energies, $P_T \gg P_T \gg m$, where $P_T$ and $m$ are the transverse momentum and mass of the secondary particle respectively. Hence, the above relation reduces to the form

$$Y \approx \eta = -\ln \tan \theta/2, \quad (5.2)$$

where $\theta$ is the space angle of the secondary particle with respect to the incident particle. The variable $\eta$ is called the pseudorapidity in the laboratory frame. It has been found (4,29) that the pseudorapidity distribution is practically the same as the true rapidity distribution but differs slightly in the low rapidity region. We have therefore analysed the data in terms of the pseudorapidity variable. In this chapter we study the rapidity distributions of shower particles produced in $^{12}\text{C-Em}$ and central $^{12}\text{C-Em}$ collisions at 4.5 A GeV/c. Central collisions are defined as events with $N_h \geq 28$ and having no observable projectile fragments, even singly charged, emitted within $3^\circ$ of the beam direction. Out of 1300, only 152 events satisfied the criteria for centrality.
Our criteria ensure that these events are due to the central collision of $^{12}$C with AgBr nuclei. Our results have been compared with those of nucleon-nucleus, hadron-nucleus and nucleus-nucleus collisions at different energies.

5.2 **Rapidity Gap Distributions**

A number of attempts have been made to understand the mechanism of multiparticle production in terms of the cluster model. According to the model, a hadron system (cluster) is produced when a nucleon collides with another nucleon which subsequently decays into final state hadrons. There are interesting predictions regarding the nature of cluster, like the droplet of gluons (30), resonances (31), excited hadronic states (32) and collective phenomenon without dynamical significance (33). But there is no conclusive evidence in favour of any of these phenomena till now. A number of workers (15,19,34-37) have suggested that the mechanism of multiparticle production through clusters can be well understood by the method of rapidity gap distributions. Quite interesting results are expected to be obtained on the formation of clusters and their decay.

It has been reported by many workers (5,9,17,38-4C) that the rapidity gap distribution in high energy collisions can be represented by two channel generalization of the Chev-Pignotti model (41). In that case the rapidity gap distribution could be written as following (12).
\[ \frac{dN}{dr} = Ae^{-Br} + Ce^{-Dr}, \] (5.3)

where \( r \) is the rapidity gap and the parameters \( A \) and \( C \) are the normalizing factors and depend on the number of particles contributing in the first and second region of the rapidity distribution respectively. The parameters \( B \) and \( D \) denote the slopes in the two parts of the rapidity distribution. The parameter \( B \) in the above equation is related to the cluster density, i.e., the number of cluster produced per unit rapidity gap. The greater the cluster density, the more closely spaced the particles are in the rapidity space and consequently the number of particles with low values of rapidity gap \( r \) is large; this increases the values of \( B \). Thus, the parameter \( B \) is a measure of the strength of correlation. The slope \( D \) in the second term signifies the independent emission of particles contributing in that part of the rapidity distribution.

To study the rapidity gap distribution between charged particles produced in high energy collisions, the rapidity, \( \eta \), for all charged secondary particles of each collision is calculated using relation (5.2) and then the rapidity of all the secondary charged particles in a collision are arranged in increasing order \( (\eta_1 < \eta_2 < \eta_3 \ldots \ldots \ldots \eta_n) \). The differences \( r(2) = \eta_{i+1} - \eta_i \), where \( i = 1, 2, \ldots \ldots \ldots n-1 \), \( (5.4) \) are calculated. This gives rise to two particles' rapidity gap distribution. Similarly, the differences \( r(3) = \eta_{i+2} - \eta_i \), where \( i = 1, 2, \ldots \ldots \ldots n-2 \), \( (5.5) \)
give three particles' rapidity gap distribution and so on.

Distributions of the rapidity difference between two neighbouring particles, observed in $^{12}\text{C}-\text{Em}$ and central $^{12}\text{C}-\text{Em}$ collisions are shown in Fig. 5.1. The presence of peaks at relatively smaller values of rapidity gaps clearly indicates the existence of strong two particle correlations. This shows that the particle production in nucleus-nucleus collisions also takes place via cluster formation. It may be mentioned here that the solid curves in the figure correspond to Eq. 5.3. The fit of the experimental data is obtained by the method of least squares fit using VAX-11 computer. The two broken curves in the figures show the independent contributions of the two exponential terms of Eq. 5.3. It is seen in the figure that a major contribution to the correlations comes from the first term of Eq. 5.3, referred to as short range correlation; while the contribution of the second term, the so called long range correlations, appears to be quite small.

Table 5.1 shows the values of correlation parameters for $\pi-\text{Em}$ collisions ($N_h \geq 28$) at 50 GeV/c (42), $p-\text{Em}$ collisions ($N_h \geq 28$) at 400 GeV/c (42), and central $^{12}\text{C}-\text{Em}$ collisions ($N_h \geq 28$) at 4.5 A GeV/c. It is clear from the table that the strength of correlation in central collisions of $^{12}\text{C}-\text{Em}$ is much higher than that $p-\text{Em}$ collisions at 400 GeV/c. It means that the mass of heavy projectiles competes with the primary energy of singly charged particles in the production of clusters. The table
Fig. 5.1 Two particle rapidity gap distributions for $^{12}\text{C}-\text{Em}$ and central $^{12}\text{C}-\text{Em}$ collisions at 4.5 $\text{A GeV/c}$. Solid lines represent Eq. 5.3 of the text.
<table>
<thead>
<tr>
<th>Projectile momentum</th>
<th>Type of collisions</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>$\chi^2$/D.G.F</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 GeV/c</td>
<td>$\pi^-$-Em ($N_h \geq 28$)</td>
<td>4.39±0.57</td>
<td>5.00±0.61</td>
<td>0.25±0.09</td>
<td>0.80±0.20</td>
<td>0.14</td>
<td>42</td>
</tr>
<tr>
<td>400 GeV/c</td>
<td>p-Em ($N_h \geq 28$)</td>
<td>4.50±0.21</td>
<td>5.00±0.11</td>
<td>0.10±0.09</td>
<td>0.80±0.19</td>
<td>0.64</td>
<td>42</td>
</tr>
<tr>
<td>4.5 A GeV/c</td>
<td>$^{12}$C-Em ($N_h \geq 28$)</td>
<td>5.06±0.58</td>
<td>8.06±1.78</td>
<td>1.38±0.54</td>
<td>2.92±0.31</td>
<td>0.91</td>
<td>Present work</td>
</tr>
</tbody>
</table>
also indicates that the strength of correlation increases with the mass of the projectile.

In order to study the dependence of the cluster size on the target mass, the experimental data of $^{12}$C-Em collisions at 4.5 A GeV/c have been divided into different groups: $N_h > 6$, $0 \leq N_h \leq 6$, $7 \leq N_h \leq 27$ and $N_h \geq 28$. Two particles' rapidity gap distributions are shown in Fig. 5.2 for different $N_h$ groups. It is interesting to see from the figure that clear peak exists at relatively smaller value of rapidity gaps in two particles rapidity gap distribution, which gives evidence for strong short-short correlations and supports cluster formation (43). It may be seen in Fig. 5.2 that these distributions are nicely reproduced by Eq. 5.3. The values of constant A, B, C and D of Eq. 5.3, along with the value of $\chi^2$/D.O.F obtained for two particle rapidity gap distributions for all $N_h$ groups are presented in Table 5.2. It may be noted from the table that the value of coefficient B, which is regarded as a measure of the strength of correlations, remains almost constant (within errors), irrespective of the value of $N_h$. This means that the strength of correlation is independent of target size. Furthermore, the fact that almost the same value of B is found at cosmic ray energies (12) indicates that the strength of correlation does not depend on the primary energy. These results, therefore, reveal that the two particles' correlations are independent of the incident energy and size of the target.
Fig. 5.2 Two particle rapidity gap distributions of different $N_h$ groups for $^{12}$C-Em collisions at 4.5 A GeV/c.
5.3 Higher Order Correlations

In order to examine the existence of higher order correlations, the three and four particles' rapidity gap distributions for $^{12}$C-Em and central $^{12}$C-Em collisions at 4.5 A GeV/c are shown in Fig. 5.3. Absence of peaks at small values of the gap indicates that higher order correlations are not present in our $^{12}$C-Em and central $^{12}$C-Em collisions at 4.5 A GeV/c.

5.4 Production of Heavy Clusters

It has been speculated that apart from light clusters consisting of a few particles, heavy fireball type clusters might be produced in high energy nuclear collisions (43). In order to check this, Adamovich et al (43) have suggested a method of analyzing high energy collisions by the investigations of all possible available distributions of rapidity intervals. The rapidity gap $n_{rk}$ between two particles, consisting k particles in between is defined as

$$n_{rk} = \eta_{i+k+1} - \eta_i ,$$

(5.6)

where $1 \leq i \leq n-k-1$, $0 \leq k \leq n-2$ and n is the total number of showers in an event. According to Adamovich et al (43), in the case of production of two heavy clusters, a two bump structure could be noticeable in rapidity gap distributions for $k \geq n/2$. The two bump structure would be more pronounced for events with
Fig. 5.3 Three and four particle rapidity gap distributions for $^{12}$C-Em and central $^{12}$C-Em collisions at 4.5 A GeV/c.
<table>
<thead>
<tr>
<th>Type of collisions</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>$\chi^2$/D.o.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_h \geq 0$</td>
<td>$3.21 \pm 0.32$</td>
<td>$7.15 \pm 0.55$</td>
<td>$1.70 \pm 0.37$</td>
<td>$2.41 \pm 0.20$</td>
<td>1.51</td>
</tr>
<tr>
<td>$0 \leq N_h \leq 6$</td>
<td>$5.67 \pm 0.51$</td>
<td>$7.95 \pm 1.01$</td>
<td>$0.67 \pm 0.27$</td>
<td>$1.89 \pm 0.32$</td>
<td>1.54</td>
</tr>
<tr>
<td>$7 \leq N_h \leq 27$</td>
<td>$3.09 \pm 0.26$</td>
<td>$6.39 \pm 1.05$</td>
<td>$1.66 \pm 0.25$</td>
<td>$2.79 \pm 0.20$</td>
<td>1.43</td>
</tr>
<tr>
<td>$N_h \geq 28$</td>
<td>$5.06 \pm 0.58$</td>
<td>$8.06 \pm 1.78$</td>
<td>$1.38 \pm 0.54$</td>
<td>$2.92 \pm 0.31$</td>
<td>0.91</td>
</tr>
</tbody>
</table>
$n_s$ value close to $\langle n_s \rangle$. In the present work $\langle n_s \rangle = (7.71 \pm 0.23)$ for $^{12}$C-Em collisions and $(15.20 \pm 1.28)$ for central $^{12}$C-Em collisions. We, therefore, selected 246 $^{12}$C-Em collisions with $n_s = 7, 8, 9$ and 48 central $^{12}$C-Em collisions with $n_s = 14, 15$ and 16. In the case of production of two heavy clusters a two bump structure has to be observed in the distributions for $k > n/2$. Figures 5.4 and 5.5 show the rapidity gap distributions for $k = 0$ to 7 and $k = 0$ to 14 for $^{12}$C-Em and central $^{12}$C-Em collisions respectively. As can be seen from the figures, there is no indication of a two bump structure in the distributions for $k > n/2$. We, therefore, conclude that heavy clusters are not produced in $^{12}$C-Em and central $^{12}$C-Em collisions at 4.5 A GeV/c. However, we have earlier obtained evidence for the production of heavy clusters in nucleon-nucleon collisions at cosmic ray energies (44). It means that the production of heavy cluster is energy dependent and our energy in the present work is below the threshold for the production of such clusters.

5.5 Conclusions

From the analysis of $^{12}$C-Em collisions at 4.5 A GeV/c we may concluded the following:

(i) The secondary particles are produced via cluster formation in nucleus-nucleus collisions as in nucleon-nucleon and nucleon-nucleus collisions and each clusters decay into at least three charged particles.

(ii) The value of the strength of correlation remains almost
Fig. 5.4 Rapidity gap distributions for $k = 0$ to 7 for $^{12}$C-Em collisions at 4.5 A GeV/c.
Fig. 5.5 Rapidity gap distributions for $k = 0$ to 14 for central $^{12}$C-$Em$ collisions at 4.5 A GeV/c.
constant irrespective of the value of $N_h$. These results, therefore, suggest that the strength of correlation is independent of the target size.

(iii) Higher order correlations are not present and there is no evidence for the production of heavy fireball type clusters in $^{12}$C-Em collisions at 4.5 A GeV/c.
REFERENCES

CHAPTER VI

PROPERTIES OF PROJECTILE FRAGMENTS FROM $^{12}_C+^{Em}$ COLLISIONS AT 4.5 A GeV/c

6.1 Introduction

The first experimental information about the fragmentation of nuclei was obtained in experiments with cosmic rays (1,2). From the study of fragmentation of nuclei one may obtain information about the internal structure of nuclei under condition of small transfer of energy and momentum. For momentum transfer just above the threshold value for the breakup of the projectile, the nucleus as a whole participates in the reaction. Thus, one may expect the observed spectra to reflect the distributions of various constituents inside the nucleus. Projectile fragments which have not experienced any strong collision tend, on the other hand, to keep various static properties that the projectile nucleus had before the collision. The projectile fragments may thus be useful in determining the momentum distribution of a nuclear cluster inside the nucleus.

Knowledge of fragmentation characteristics of nuclei is required for solution of a number of problems of astrophysics, cosmic ray physics and radiation physics. The production of beams of relativistic nuclei in the accelerators at Dubna and Berkeley has made it possible to obtain quantitative information on this question which is considerably more accurate than that obtained
previously in experiments with cosmic rays. The first experiments on investigation of fragmentation of certain light relativistic nuclei in accelerators were carried out at momentum $2.9 \text{ A GeV/c}$ by means of a spectrometer (3-5) and nuclear emulsion (6-9). In electronic experiments (3-5) in which projectile fragments with an emission angle $\Theta < 0.7^\circ$ were detected, the fragmentation cross-section was found to factor with a projectile and target related part (3,4,10). Electronic experiments have great advantage for obtaining accurate quantitative information on production cross-sections and momentum characteristics of projectile fragments. They are characterized by high resolution in mass and charge of the fragments and by high statistics. On the other hand, it should be noted that in experiments carried out by means of electronics only a very narrow spatial cone, around the direction of motion of the primary nucleus, $\Theta < 12.5 \text{ mrad}$ has been studied, i.e. the total cross-sections have not been measured. Another fundamental difficulty of electronic experiments is the unobservability of collision events, which greatly limits the possibilities of study of the dynamics of nucleus-nucleus collisions as a whole, in particular of various correlations between the fragmentation of the two nuclei and the production of particles. Therefore, track devices have an important role in the study of high energy nucleus-nucleus collisions. Since nuclear emulsions have a $4\pi$ recording capability for all secondary particles emitted in the collision, one may have an extra advantage in studying the projectile fragmentation with
nuclear emulsion. For example, one can study collisions involving varying degrees of target excitations. It is also possible to have access to various rapidity regions and to examine the features associated with specific regions.

The geometrical aspects of the fragmentation of a nucleus can be understood in terms of the participant-spectator model (11,12). According to this model, at finite impact parameter, three regions are produced after a collision between two nuclei: The participant region, the projectile spectator and the target spectator. The projectile spectator decays mainly into nuclear clusters. Since very little momentum transfer is required to form these fragments, the projectile fragments may thus be useful in determining the momentum distribution of a nuclear cluster inside the projectile nucleus. At relativistic energies, the separation in rapidity between projectile and target fragments is large, \( \gtrsim 1 \) unit of rapidity. Consequently, no correlations exist between projectile and target nucleus and the modes of fragmentation are independent of target mass. The fragmentation cross-sections can thus be factorized into a target and projectile related parts. If we write the reaction as \( B+T = F+X \), the factorization can be expressed as \( \sigma_{BT}^F = \gamma_B^F \gamma_T \), where \( \gamma_B^F \) depends only on the projectile and the fragment and \( \gamma_T \) depends on the target. A dependence of the form \( \gamma_T \propto T^{-0.25} \) has been found. In inclusive experiments (4) in which fragments (\( \theta < 0.7^\circ \)) of light nuclei were detected with a spectrometer and there was no restriction on the degree of target
excitation, the factorization has been found to be valid. In emulsion experiments, the angular distributions of projectile fragments for events exhibiting either no or very small target excitation, exhibited features of limiting fragmentation (13,14). Exceptions to strict factorization have however been observed for fragmentation reactions in hydrogen (4) and for heavy targets where single nucleon stripping is increased by the coulomb dissociation of carbon and oxygen projectiles in the virtual photon field of the target nucleus (15). Also correlations between the average angle of emission of helium fragments and target particle multiplicity have been observed, implying that helium cross-section cannot be factorized (16).

In the present chapter, our investigation is devoted to study the fragmentation characteristics of $^{12}$C nuclei in emulsion at primary momentum $p_0 = 4.5$ A GeV/c. Multiplicities of projectile fragments of different charges have been obtained in different ensembles of collision. The dependence of the multiplicity of projectile fragments on the mass of the projectile is investigated. Our data indicate that the principle of factorization has only a limited region of applicability. To test the validity of limiting fragmentation hypothesis, the projected angular distribution and momentum distributions have been studied in detail. Azimuthal correlations have also been studied for $Z \geq 2$ fragments. Finally, we study the anomalous behaviour of $Z = 2$ fragments emitted from $^{12}$C-Em collisions at 4.5 A GeV/c.
6.2 Multiplicity of Projectile Fragments

The factorization of cross-section observed by the Berkeley group (3,5,10) in electronic experiments implies that the charge composition of projectile fragments does not depend on the mass of the target. Data on multiplicities of fragments with different charges in different target groups of $^{12}$C-Em collisions are presented in Tables 6.1 and 6.2. The errors quoted are statistical. The data show that for an emulsion experiment, this statement is not correct. It can be noted from the tables that the average multiplicity decreases as the charge of the fragment increases. The multiplicities of fragments of any charge decrease with increasing mass of the target. Thus the composition of fragments depends considerably on the mass of the target, an explicit violation of the principle of factorization. It is observed by Berkeley group that the ratios of differential cross-sections for production of fragments near $^{0}$C in different targets are constant and approximately equal to the ratios of the geometrical cross-sections. Results presented in Table 6.3 again contradict this statement. In fact, the results indicate that there is a considerable dependence of the cross-section on the mass of the target.

Figure 6.1 shows the multiplicity distributions of $Z = 2,3$ and $\geq 4$ fragments. Our multiplicity distributions are in agreement with the corresponding distributions from $^{14}$N-Em collisions at 2.1 A GeV (17). Hence, one can say that within this short range of energy, the projectile fragmentation is energy independent.
Fig. 6.1 The multiplicity distribution of projectile fragments of charge $Z = 2, 3$ and $\geq 4$. 
Table 6.1

Average multiplicities of projectile fragments produced in $^{12}\text{C}$-Em collisions at 4.5 A GeV/c

<table>
<thead>
<tr>
<th>Charge of fragments</th>
<th>$^{12}\text{C}$-H</th>
<th>$^{12}\text{C}$-CNO</th>
<th>$^{12}\text{C}$-AgBr</th>
<th>$^{12}\text{C}$-Em</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = 2$</td>
<td>1.38 ± 0.09</td>
<td>0.84 ± 0.04</td>
<td>0.22 ± 0.02</td>
<td>0.66 ± 0.02</td>
</tr>
<tr>
<td>$Z = 3$</td>
<td>0.09 ± 0.02</td>
<td>0.06 ± 0.01</td>
<td>0.022 ± 0.01</td>
<td>0.05 ± 0.01</td>
</tr>
<tr>
<td>$Z = 4$</td>
<td>0.06 ± 0.02</td>
<td>0.04 ± 0.01</td>
<td>0.014 ± 0.01</td>
<td>0.03 ± 0.01</td>
</tr>
<tr>
<td>$Z = 5$</td>
<td>0.02 ± 0.01</td>
<td>0.03 ± 0.01</td>
<td>0.012 ± 0.01</td>
<td>0.022 ± 0.01</td>
</tr>
<tr>
<td>$Z = 6$</td>
<td>0.02 ± 0.01</td>
<td>0.02 ± 0.01</td>
<td>0.009 ± 0.01</td>
<td>0.02 ± 0.01</td>
</tr>
</tbody>
</table>
Further we have not found even a single event emitting two fragments with $Z = 3$. Thus the upper limit for the production cross-section of such events is $7.7 \times 10^{-4}$ of the total reaction cross-section. Whereas the values reported by Jakobsson et al (18) and Judek et al (19) are $3.7 \times 10^{-3}$ and $1.3 \times 10^{-3}$ respectively.

Our results on the fragmentation of carbon nuclei at 4.5 A GeV/c show that the principle of factorization observed in electric experiments for fragments emitted near $0^\circ$ has a restricted region of applicability. It is broken in an emulsion experiment where the total cross-sections are measured. A similar result has been observed in case of other projectiles (20).

It would be interesting to investigate the dependence of the average multiplicities of fragments on the mass of the projectile. In Fig. 6.2 we plot $\langle N_z \rangle$ vs $A$, the mass number of the projectile for fragments with charge $Z = 1, 2$ and $Z \geq 3$. Data have been taken from references (20-23). An expression of the type $\langle N_z \rangle = \text{Const.} A^\alpha$ can well describe the dependence. The best fit values of $\alpha$ are $(0.75 \pm 0.08)$, $(0.50 \pm 0.26)$ and $(1.20 \pm 0.22)$ for fragments of charge $Z = 1, 2$ and $\geq 3$ respectively.

6.3 Angular Distribution of Projectile Fragments

The mechanism of high energy heavy ion collisions, from a simple geometrical point of view, may well be understood in terms of participant-spectator model (24-26). This model defines the spectators of the projectile as projectile fragments emitted at $0^\circ$.
Fig. 6.2 The dependence of average multiplicity of projectile fragments of charge $Z = 1, 2$ and $Z \geq 3$ on the mass of the projectile, $A$. The solid line represents the relation $\langle N_Z \rangle = \text{Const.} \, A^\alpha$ (See Text).
### Table 6.2

<table>
<thead>
<tr>
<th>Multiplicity ratio</th>
<th>Target nucleus</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>CNC</td>
<td>AgBr</td>
</tr>
<tr>
<td>$&lt;N_z = 2&gt;$</td>
<td>1.98 ± 0.18</td>
<td>0.86  ± 0.04</td>
<td>0.28 ± 0.04</td>
</tr>
<tr>
<td>$&lt;N_z = 1&gt;$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$&lt;N_z &gt;= 3&gt;$</td>
<td>0.27 ± 0.03</td>
<td>0.16 ± 0.02</td>
<td>0.08 ± 0.01</td>
</tr>
<tr>
<td>$&lt;N_z = 1&gt;$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$&lt;N_z &gt;= 3&gt;$</td>
<td>0.14 ± 0.02</td>
<td>0.19 ± 0.03</td>
<td>0.26 ± 0.04</td>
</tr>
<tr>
<td>$&lt;N_z = 2&gt;$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 6.3

The ratio of the average multiplicities of projectile fragments for different target combinations

<table>
<thead>
<tr>
<th>Charge of fragment $Z$</th>
<th>$&lt;N_z &gt;$ CNO</th>
<th>$&lt;N_z &gt;$ AgBr</th>
<th>$&lt;N_z &gt;$ AgBr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$&lt;N_z &gt;$ H</td>
<td>$&lt;N_z &gt;$ H</td>
<td>$&lt;N_z &gt;$ CNO</td>
</tr>
<tr>
<td>1</td>
<td>1.35 ± 0.11</td>
<td>1.08 ± 0.09</td>
<td>0.80 ± 0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.61 ± 0.04</td>
<td>0.15 ± 0.02</td>
<td>0.26 ± 0.03</td>
</tr>
<tr>
<td>≥ 3</td>
<td>0.79 ± 0.12</td>
<td>0.29 ± 0.05</td>
<td>0.37 ± 0.09</td>
</tr>
</tbody>
</table>
with the same velocity as that of the projectile and having a mass less than that of the projectile. Further, the spectator breakup properties are expected to be independent of what happens in the participant and depend only on the collision geometry (27).

The projected angular distributions of all the identified fragments with \( Z \geq 2 \) appear in Fig. 6.3. It is clear from the figure that almost all the projectile fragments are confined to a narrow forward cone with a maxima at 0°, which agrees well with the predictions of the participant–spectator model. Since the forward cone of 2° covers more than 98% of the fragments emitted, all our detailed analysis is confined to this region.

Lepore and Riddell (28) have studied the fragmentation of relativistic nuclei using the sudden approximation and shell model functions. They have shown that the momentum distribution of fragments in the rest frame of projectile is approximately Gaussian and the width of the distribution is given by

\[
\sigma^2(P) = \left[ m \omega A_F (A_p - A_F) / 2A_p \right] (\text{MeV})^2, \quad (6.1)
\]

where \( m \) is the proton mass, \( A_p \) is the mass number of the projectile, \( A_F \) is the mass number of the fragment and \( \omega = 45 A_p^{-1/3} - 25 A_F^{-2/3} \). The width of the corresponding projected angular distribution in the laboratory frame can be obtained using the following relation

\[
\sin \sigma(\theta_P) = \frac{\sigma(P) A_p}{P_0 A_F}, \quad (6.2)
\]
Fig. 6.3 Projected angular distribution of $Z \geq 2$ fragments with Gaussian curve fitted to the data for $\Theta_p \leq 2^\circ$. 

$Z \geq 2$

$\sigma = 0.56 \pm 0.02$

$\chi^2 / D.O.F. = 2.11$
where $P_o$ is the projectile momentum. From Eqs. 6.1 and 6.2, it is clear that the width of the momentum or projected angular distribution is independent of target mass.

The projected angular distributions for $Z = 2$ fragments for $^{12}\text{C}-\text{H}$, $^{12}\text{C}-\text{CNO}$, $^{12}\text{C}-\text{AgBr}$ and $^{12}\text{C}-\text{Em}$ collisions are shown in Fig. 6.4, along with the fitted Gaussian curves for $\Theta_p \leq 2^\circ$. A Gaussian curve of the form $N(\Theta_p) = A \exp\left(-\Theta_p^2/2\sigma^2\right)$ was fitted to each of these distributions. The values of $\sigma$ are also shown in the figures. Figure 6.5 shows the projected angular distributions of $Z = (3-5)$ fragments. Due to low statistics of these fragments, distributions for $^{12}\text{C}-(\text{H+CNO})$ and $^{12}\text{C}-\text{Em}$ have been plotted. The values of standard deviations of the fitted Gaussian curves are also shown in the figures. It is evident from the figures that the distribution becomes narrower as the charge of the fragment increases.

Table 6.4 present the summary of results on the projected angular distributions of projectile fragments. Also presented in the table are $\sigma$ values calculated using Eq. 6.1. It can be noted from the table that $\sigma$ values are almost independent of target mass, except in case of $Z = 2$ fragments for which a weak target mass dependence is observed. These values for different fragments are also comparable within statistical errors to the theoretical values obtained using Eq. 6.1. Thus, we can argue that our results are consistent with the characteristic features expected from the limiting fragmentation which implies that both projectile and
Fig. 6.4 Projected angular distributions of $Z = 2$ fragments in different target groups with Gaussian curves fitted to the data for $\Theta_p \leq 2^\circ$. 

$^{12}\text{C} - \text{Em}$  
$\sigma = 0.59 \pm 0.02$  
$\chi^2/\text{D.O.F.} = 1.23$

$^{12}\text{C} - \text{H}$  
$\sigma = 0.54 \pm 0.02$  
$\chi^2/\text{D.O.F.} = 0.54$

$^{12}\text{C} - \text{AgBr}$  
$\sigma = 0.69 \pm 0.05$  
$\chi^2/\text{D.O.F.} = 0.31$

$^{12}\text{C} - \text{CNO}$  
$\sigma = 0.58 \pm 0.02$  
$\chi^2/\text{D.O.F.} = 1.04$
Fig. 6.5 Projected angular distributions of multi-charged fragments \( Z = (3-5) \) with Gaussian curves fitted to the data for \( \Theta_p \leq 1^\circ \).
Table 6.4
Standard deviations of projected angular distributions of projectile fragments from $^{12}\text{C}-\text{Em}$ and $^{14}\text{N}-\text{Em}$ collisions.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Ensemble</th>
<th>He</th>
<th>Li</th>
<th>Be</th>
<th>B</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$</td>
<td>$^{12}\text{C}$-H</td>
<td>0.54 ± 0.02</td>
<td>0.37 ± 0.04</td>
<td>0.25 ± 0.04</td>
<td>0.17 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^{12}\text{C}$-CNO</td>
<td>0.58 ± 0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$-AgBr</td>
<td>0.69 ± 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Present work</td>
</tr>
<tr>
<td>$^{12}\text{C}$-Em</td>
<td>0.59 ± 0.02</td>
<td>0.37 ± 0.04</td>
<td>0.29 ± 0.07</td>
<td>0.18 ± 0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td>0.46</td>
<td>0.31</td>
<td>0.22</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>$^{14}\text{N}$-h ≥ 0</td>
<td>0.71 ± 0.03</td>
<td>0.71 ± 0.05</td>
<td>0.53 ± 0.05</td>
<td>0.35 ± 0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^{14}\text{N}$-h ≤ 8</td>
<td>0.68 ± 0.03</td>
<td>0.53 ± 0.06</td>
<td>0.43 ± 0.04</td>
<td>0.25 ± 0.03</td>
<td>14</td>
</tr>
<tr>
<td>Theory</td>
<td>0.73</td>
<td>0.47</td>
<td>0.39</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
target are fragmented independent of each other. Bhania et al (14) studied the fragmentation of $^{14}$N nuclei at 2.8 A GeV/c and observed that the features of projected angular distributions of fragments from event with $C \leq N_h \leq 8$ ( peripheral collisions) are consistent with the hypothesis of limiting fragmentation. Whereas our results on the projected angular distribution of fragments from the fragmentation of $^{12}$C nuclei at 4.5 A GeV/c show that the hypothesis of limiting fragmentation is valid for peripheral as well as quasi-central collisions. Thus, the domain of validity of the limiting fragmentation hypothesis extends as the energy of the projectile nucleus increases. However, it is worth mentioning here that the analysis of projected angular distribution was restricted to $\theta \leq 2^0$ for $Z = 2$ fragments and $\theta \leq 1.0^0$ for $Z \geq 3$ fragments. There are quite a few fragments, especially of $Z = 2$, which lie outside the Gaussian tail of the distribution.

6.4 Transverse Momentum Distribution

The transverse momentum distribution of projectile fragments have been studied by many workers (8,9,14,20,29,30). It has been found that the distribution could be described by a Gaussian curve in the rest frame of the projectile nucleus and the standard deviation of the distribution has a parabolic dependence on the mass of the fragment. These features of the transverse momentum distribution follow from the statistical approach to the fragmentation process which admits no correlations between momenta of intra-nuclear nucleons. However, deviation from Gaussian distri-
butions have been observed in some experiments.

In an emulsion experiment it is not possible to make direct measurement of momentum of high energy projectile fragments. However, the transverse momentum can indirectly be measured by using the fact that the fragments have nearly the same momentum per nucleon as that of the projectile. Thus the transverse momentum of a fragment of charge $Z$ can be calculated by using the relation

$$p_t = A_F P_0 \sin \theta,$$  \hspace{1cm} (6.3)

where $P_0$ is the momentum of the projectile, $A_F$ is the mass number of the fragment and $\theta$ is the angle of emission of the fragment. For fragments with $Z \leq 2$ the above relation gives a reliable estimate of the transverse momentum. However, due to the excess of neutron rich isotopes, it gives a lower limit for the transverse momentum of fragments with $Z \geq 3$.

Table 6.5 gives the average transverse momentum, $\langle p_t \rangle$, of fragments with different charges in collisions of $^{12}$C with different target groups. We notice that $\langle p_t \rangle$ increases with the mass of the target. Figures 6.6 and 6.7 show the transverse momentum distributions of fragments with charge $Z \geq 2$. For $Z = 2$ fragments these distributions are plotted for different target groups. The distributions are fitted with a curve of the type

$$N(p_t) = A p_t \exp\left(-\frac{p_t^2}{2\sigma^2}\right),$$  \hspace{1cm} (6.4)
Fig. 6.6 Transverse momentum distributions of $Z = 2$ fragments in different target groups with Gaussian curves fitted to the data for $p_t \leq 500$ MeV/c.

For $^{12}$C - Em:
- $\sigma = 159 \pm 2$
- $X^2$/D.O.F. = 1.32

For $^{12}$C - H:
- $\sigma = 159 \pm 5$
- $X^2$/D.O.F. = 0.48

For $^{12}$C - CNO:
- $\sigma = 166 \pm 5$
- $X^2$/D.O.F. = 0.98

For $^{12}$C - AgBr:
- $\sigma = 167 \pm 12$
- $X^2$/D.O.F. = 0.93
Fig. 6.7 Transverse momentum distributions of Z = 3, 4 and 5 fragments with Gaussian curves fitted to data for $P_t \leq 500$ MeV/c.
### Table 6.5

Average transverse momenta of projectile fragments in different group of $^{12}\text{C-Em}$ collisions

<table>
<thead>
<tr>
<th>Charge of fragments</th>
<th>$^{12}\text{C-H}$</th>
<th>$^{12}\text{C-CNO}$</th>
<th>$^{12}\text{C-AgBr}$</th>
<th>$^{12}\text{C-Em}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = 2$</td>
<td>0.202 ± 0.01</td>
<td>0.237 ± 0.01</td>
<td>0.303 ± 0.02</td>
<td>0.235 ± 0.01</td>
</tr>
<tr>
<td>$Z = 3$</td>
<td>0.239 ± 0.05</td>
<td>0.289 ± 0.05</td>
<td>0.381 ± 0.09</td>
<td>0.297 ± 0.04</td>
</tr>
<tr>
<td>$Z = 4$</td>
<td>0.202 ± 0.05</td>
<td>0.227 ± 0.06</td>
<td>0.299 ± 0.06</td>
<td>0.241 ± 0.04</td>
</tr>
<tr>
<td>$Z = 5$</td>
<td>0.207 ± 0.09</td>
<td>0.292 ± 0.10</td>
<td>0.355 ± 0.12</td>
<td>0.295 ± 0.06</td>
</tr>
<tr>
<td>$Z = 6$</td>
<td>0.216 ± 0.08</td>
<td>0.348 ± 0.12</td>
<td>0.325 ± 0.12</td>
<td>0.304 ± 0.07</td>
</tr>
</tbody>
</table>
for \( p_t \leq 500 \) MeV/c. The above distribution is expected if each component of transverse momentum, \( P_x \) and \( P_y \), follows a Gaussian distribution

\[
N(p) = \lambda \exp\left(-\frac{p^2}{2\sigma^2}\right). \tag{6.5}
\]

It should be mentioned here that if we include fragments with \( p_t > 500 \) MeV/c, then the \( p_t \) distribution cannot be fitted with the curve given by Eq. 6.4.

The observed values of \( \sigma(P) \) can be related to the nuclear Fermi momentum, \( P_F \), assuming the sudden emission of \( \alpha \)-clusters \((31,32)\). The relation comes out to be

\[
\sigma^2(P) = \frac{P_F^2}{5} \frac{A_F(A_P - A_F)}{(A_P - 1)^2}. \tag{6.6}
\]

and if it is assumed that the nucleus comes to thermal equilibrium, then \( \sigma(P) \) can also be related to excitation energy, \( KT \), through the relation

\[
\sigma^2(P) = KT \frac{m(A_P - A_F)}{\hat{A}_P}, \tag{6.7}
\]

where \( A_P \) and \( A_F \) are respectively the mass of the projectile and the fragment, and \( m \) is the proton mass.

Table 6.6 presents values of \( \sigma(p) \) for fragments of different charges observed in the present as well as in other experiments on the fragmentation of relativistic nuclei along with the values of \( P_F \) and \( KT \) calculated using relations 6.6 and 6.7. We notice that
Table 6.6

Table showing the temperature (excitation energy or binding energy), Fermi momentum and $\sigma(p)$

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Fragments</th>
<th>$\sigma(p)$ (Experimental)</th>
<th>$\sigma(p)$ (Theoretical)</th>
<th>Fermi momentum ($p_F$) MeV/c</th>
<th>Temperature (K) MeV</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$ (1 GeV/c)</td>
<td>He</td>
<td>125 ± 3</td>
<td>143</td>
<td>176 (221)*</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>Li</td>
<td>122 ± 10</td>
<td>146</td>
<td>151</td>
<td></td>
<td>5.3</td>
<td>5</td>
</tr>
<tr>
<td>Be</td>
<td>131 ± 9</td>
<td>125</td>
<td>187</td>
<td></td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>135 ± 9</td>
<td>108</td>
<td>224</td>
<td></td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$ (2.85 GeV/c)</td>
<td>He</td>
<td>129 ± 1</td>
<td>134</td>
<td>169 (221)*</td>
<td>6.7</td>
<td>5</td>
</tr>
<tr>
<td>Li</td>
<td>127 ± 7</td>
<td>146</td>
<td>157</td>
<td></td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>133 ± 3</td>
<td>125</td>
<td>190</td>
<td></td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>134 ± 3</td>
<td>108</td>
<td>222</td>
<td></td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$ (4.5 GeV/c)</td>
<td>He</td>
<td>164 ± 8</td>
<td>134</td>
<td>215 (221)*</td>
<td>10.8</td>
<td>Present Experiment</td>
</tr>
<tr>
<td>Li</td>
<td>215 ± 21</td>
<td>146</td>
<td>266</td>
<td></td>
<td>16.4</td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>161 ± 16</td>
<td>125</td>
<td>230</td>
<td></td>
<td>12.3</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>132 ± 59</td>
<td>108</td>
<td>219</td>
<td></td>
<td>11.2</td>
<td></td>
</tr>
<tr>
<td>$^{14}\text{N}$ (2.8 GeV/c)</td>
<td>He</td>
<td>141 ± 6</td>
<td>134</td>
<td>180 (226)*</td>
<td>7.4</td>
<td>14</td>
</tr>
<tr>
<td>Li</td>
<td>212 ± 15</td>
<td>153</td>
<td>247</td>
<td></td>
<td>13.9</td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>237 ± 22</td>
<td>152</td>
<td>285</td>
<td></td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>174 ± 20</td>
<td>131</td>
<td>221</td>
<td></td>
<td>11.3</td>
<td></td>
</tr>
</tbody>
</table>

* Fermi momentum obtained in electron scattering experiment of Moniz et al (33).
\( \sigma(P) \) values observed in emulsion experiments are higher than those observed in \( C^0 \) experiment (5). This is due to the fact that in the \( C^0 \) experiment projectile fragments with \( \Theta < 0.7^0 \) were detected using a spectrometer and therefore large transverse momentum transfers were not recorded in that experiment. The values of Fermi momentum obtained in the present experiment are quite comparable with those obtained in electron scattering experiment of Moniz et al (33). We further notice that the values of the excitation energy \( K_T \) are also comparable to the binding energy per nucleon, indicating that very small energy transfer takes place between the target and a fragment during the fragmentation process.

6.5 Azimuthal Correlations

In section (6.4) we studied the transverse momentum distributions of fragments and it was observed that the presence of a high \( p_t \) tail distorts the distribution and increases \( \langle p_t \rangle \) of fragments. This may be due to the transverse motion and/or the angular momentum of the fragmenting projectile spectator. These features of the projectile spectator can be detected by studying the azimuthal correlations among the fragments.

A study of correlations in the azimuthal plane can give useful information on the mechanism of production; in particular, high angular momentum transferred to the produced particles will lead to large values of the coplanarity coefficients. For example, a transverse motion of angular momentum of the projectile spectator
may lead to non-zero value of coefficient of asymmetry \( r \) of coplanarity \( B \). The coefficients are defined as following:

\[
A = \left( \int_{\pi/2}^{\pi} \frac{d\sigma}{d\varepsilon} \, d\varepsilon - \int_{0}^{\pi/2} \frac{d\sigma}{d\varepsilon} \, d\varepsilon \right) / \int_{0}^{\pi} \frac{d\sigma}{d\varepsilon} \, d\varepsilon , \quad (6.8)
\]

\[
B = \left( \int_{0}^{\pi/4} \frac{d\sigma}{d\varepsilon} \, d\varepsilon - \int_{\pi/4}^{3\pi/4} \frac{d\sigma}{d\varepsilon} \, d\varepsilon + \int_{3\pi/4}^{\pi} \frac{d\sigma}{d\varepsilon} \, d\varepsilon \right) / \int_{0}^{\pi} \frac{d\sigma}{d\varepsilon} \, d\varepsilon , \quad (6.9)
\]

where \( \varepsilon \) is the angle between the transverse momenta of the fragments and can be calculated using the relation

\[
\varepsilon_{ij} = \cos^{-1} \left( \frac{\vec{P}_{it} \cdot \vec{P}_{jt}}{|\vec{P}_{it} \cdot \vec{P}_{jt}|} \right) , \quad (6.10)
\]

In Table 6.7 we present the values of \( A \) and \( B \) for fragments with \( Z = 2 \) and \( Z \geq 3 \) produced in collisions of \(^{12}\text{C}\) with different target groups. We find that there exist, not large, but statistically significant azimuthal correlations among the projectile fragments. This indicates that the fragmenting nucleus gets a transverse momentum during the collision.

### 6.6 Mean Free Path of \( Z = 2 \) Fragments

A number of experiments using emulsion evidencing a short mean free path component among relativistic projectile fragments of high energy cosmic ray nuclei have been reported sporadically since 1954. These fragments of short mean free path are called
Table 6.7

Values of $A$ and $B$ in various subensembles of events

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}$C-H</td>
<td>$0.065 \pm 0.009$</td>
<td>$0.011 \pm 0.004$</td>
</tr>
<tr>
<td>$^{12}$C-CNO</td>
<td>$-0.045 \pm 0.008$</td>
<td>$-0.029 \pm 0.006$</td>
</tr>
<tr>
<td>$^{12}$C-AgBr</td>
<td>$-0.049 \pm 0.010$</td>
<td>$0.041 \pm 0.010$</td>
</tr>
<tr>
<td>$^{12}$C-Em</td>
<td>$-0.055 \pm 0.006$</td>
<td>$-0.002 \pm 0.001$</td>
</tr>
<tr>
<td>$Z \geq 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}$C-Em</td>
<td>$-0.018 \pm 0.007$</td>
<td>$0.044 \pm 0.010$</td>
</tr>
</tbody>
</table>
anomalons. The first evidence for anomalous extranuclear cascading induced by heavy primary cosmic ray nuclei was seen in photographic nuclear research emulsion by Milone (34) and subsequently confirmed by others (35-37). In 1959 Friedlander and Spirchez (38) examined six cosmic ray initiated cascades and found a difference between the mean free path of first and second generation fragments. Some examples of these are schematically shown in Fig. 6.8.

The first detailed and systematic study of anomalons was performed by Judek (37,39) in emulsions exposed to cosmic rays. On the basis of mean free path measurements of relativistic cosmic ray primary and secondary nuclei, Judek concluded that a few percent of the secondary nuclei with charges $1 \leq Z \leq 4$ had anomalous mean free path of the order of 3 cm and that the star produced by the anomalous nuclei had the characteristics of ordinary nuclear collisions as observed in nuclear emulsion. However, more systematic studies led to mixed conclusions. Freier and Waddington (40) were unable to confirm the existence of anomalons while Cleghorn (41) and Barber et al (42) confirmed the anomalous behaviour of energetic projectile fragments in cosmic ray data. It should be mentioned here that these cosmic ray experiments allowed no control over the flux, the energy or even the type of projectile nuclei entering the detecting medium and statistics were also very small. Therefore, the above stated results were never widely recognized nor accepted. The situation rapidly changed with the availability of relativistic heavy ion beams from Berkeley Bevatron and CERN
Fig. 6.8 Multi chain events from Cosmic ray studies.
Synchrophasotron. Controlled high statistics experiments are possible with such beams and also various types of detectors may be employed.

In 1972 Judek (39) exposed an emulsion stack to 1.8 A GeV $^{16}\text{O}$ beam and confirmed her earlier result on anomalons. Later Friedlander et al (43) carried out a systematic study of projectile fragments from $^{16}\text{O}$ and $^{56}\text{Fe}$ collisions at 2.0 A GeV and suggested that 6% of the projectile fragments are anomalons with an interaction mean free path of 2.5 cm. Since then a large number of workers have also confirmed the anomalous behaviour of projectile fragments (44-52). However, in recent years a number of investigators using emulsion (53-58), Cerenkov detector (59-61) and plastic detector (62,63) have contradicted the observation of projectile fragments of anomalous mean free path.

There has been a number of theoretical attempts also to explain the existence of anomalons. Some of these suggestions are conventional, making use of the well established ideas in nuclear and particle physics (64-66) while others are highly exotic, explaining the phenomenon in terms of colour polarization of quarks (67).

In view of the present experimental situation, it can be said that the problem of anomalon is still not closed. Thus it would be interesting to study the anomalous behaviour of projectile fragments. In the following we study the mean free path of fragments of charge two emitted in $^{12}\text{C}$-Em collisions at 4.5 A GeV/c.
We used 255C $^{12}$S-Em collisions from which 1625 fragments of charge two were emitted within 3° of the beam direction. Each fragment was followed until it interacted or left the stack. The following was done by two different persons to avoid any kind of biasing. A collision was accepted only if at least one additional track was emitted. In this way we found 650 collisions of the $Z = 2$ fragments.

The mean free path of the $Z = 2$ fragments was calculated as a function of distance from the collision from which they were emitted. The tracks were divided into successive 1 cm intervals. All the tracks segments lying within the same interval were added together and divided by the total number of collisions observed in that interval. For a homogeneous beam of nuclei of charge $Z$, the mean free path $\lambda_z$ is defined via the distribution of collision distance $X$ as

$$f(x) \, dx = \exp\left(\frac{-x}{\lambda_z}\right) \, dx / \lambda_z.$$  \hspace{1cm} (5.11)

The collision mean free path is determined by $\lambda_z = \sum S_i / N$, where $S_i$ is the total length of both the interacting and non-interacting tracks followed in the $i$th interval and $N$ is the total number of collisions in that interval. In Fig. 6.9(a) we plot the values of the mean free path for the $Z = 2$ fragment as a function of distance $D$ from the interaction vertex. The dotted line shows the average value of the mean free path which is $(20.09 \pm 0.79)$ cm. As can be seen from the figure, there is no
indication of a shorter mean free path in the first few centimeters of the interaction vertex (68).

With a view to finding whether the impact parameter has any influence on the mean free path, we divide the Z = 2 fragments into two categories: (a) Fragments originating in collisions with $N_h \leq 1$ and (b) fragments originating in collisions with $N_h > 1$. In Fig. 6.9(b,c) we plot the mean free path of Z = 2 fragments belonging to the two categories. As can be seen from the figures, there is no evidence for anomalous fragments in either case (68).

Judek (69) observed a dependence of the mean free path on the emission angle $\Theta$ for Z = 1 projectile fragments. In order to see whether a similar dependence exists for the Z = 2 fragments also, we divide the data into two interval of $\Theta$: $\Theta \leq 1$ degree and $\Theta > 1$ degree. In Fig. 6.10 we plot the mean free path in the two intervals of $\Theta$ as a function of distance from the interaction vertex. We do not find any dependence on the mean free path on the emission angle (68).

Recently Bayman and Tang (70) suggested that the presence of isotope $^6 \text{He}$ may cause the appearance of an anomalous behaviour in the mean free path of Z = 2 fragments. $^6 \text{He}$ is a particle stable system with a half life of 0.8 sec and is the isobaric analogue of the 3.56 MeV, $T = 1$ excited state of Li. Its mean free path in emulsion is 12.6 cm compared to about 20 cm of $^4 \text{He}$. The large difference between the mean free path of $^6 \text{He}$ and $^4 \text{He}$ implies that the mean free path of fragments containing $^6 \text{He}$ and $^4 \text{He}$ will vary
Fig. 6.9 Mean free path versus distance from the production point for the Z = 2 fragment from collisions with (a) $N_h \geq 0$, (b) $N_h \leq 1$ and (c) $N_h > 1$. The dashed lines represent the average value.
Fig. 6.10 Mean free paths versus distance from the production point for the Z = 2 fragments emitted in different intervals of $\theta$. The dashed lines represent the average value.
with distance from the interaction vertex. Bayman and Tang further suggested that during peripheral collisions of $^{12}_C$ nuclei with emulsion, $^{12}_C$ could be excited to a $^6_He + ^6_Be$ binary cluster system which decays into one $^5_He$, one $^4_He$ and two protons. In a view to testing this hypothesis, we divide $^{12}_C$-Em collisions into the following channels.

(a) 3 x He channel - The projectile breaks up into three $Z = 2$ fragments.

(b) 2 x He channel - The projectile breaks up into two $Z = 2$ fragments and singly charged particles.

(c) 1 x He channel - The projectile breaks up into one $Z = 2$ fragment and singly charged particles.

(d) 1 x He + $F_{Z \geq 3}$ channel - The projectile breaks up into one $Z = 2$ and $Z = 3$ or 4 fragment.

If during peripheral collisions of $^{12}_C$ nuclei with emulsion the binary cluster system ($^6_He + ^6_Be$) is produced, it would manifest itself in channel (b) and therefore this channel must exhibit anomalous behaviour of $Z = 2$ fragments. Table 6.3 presents the percentage of occurrence of different reaction channels for different target groups. We notice that the percentage of occurrence of 2 x He channel is lower than that of 1 x He channel for all types of events, except for $N_H \leq 1$ events for which it is higher for 2 x He channel. It means that during peripheral collisions of $^{12}_C$ nuclei with emulsion most of the $Z = 2$ fragments are produced.
Table 6.8

Percentage of occurrence of different reaction channels

<table>
<thead>
<tr>
<th>Reaction channel</th>
<th>$^{12}$C-H</th>
<th>$^{12}$C-CNO</th>
<th>$^{12}$C-AgBr</th>
<th>$^{12}$C-Im</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x He</td>
<td>2.2 ± 0.30</td>
<td>0.9 ± 0.19</td>
<td>0.1 ± 0.07</td>
<td>3.2 ± 0.36</td>
</tr>
<tr>
<td>2 x He</td>
<td>7.5 ± 0.56</td>
<td>7.4 ± 0.56</td>
<td>2.1 ± 0.29</td>
<td>17.1 ± 0.86</td>
</tr>
<tr>
<td>1 x He</td>
<td>3.7 ± 0.39</td>
<td>10.1 ± 0.66</td>
<td>6.5 ± 0.52</td>
<td>20.4 ± 0.98</td>
</tr>
<tr>
<td>1xHe+F$_{2}$&gt;3</td>
<td>1.5 ± 0.24</td>
<td>0.9 ± 0.19</td>
<td>0.5 ± 0.14</td>
<td>2.9 ± 0.34</td>
</tr>
</tbody>
</table>
via 2 x He channel. The channel 1 x He + $F_{Z=3}$ is excluded from the analysis due to its low statistics. In Fig. 5.11 we plot the mean free path of $Z = 2$ fragments as a function of distance $D$ from the interaction vertex for different channels. No dependence of the mean free path on the distance $D$ is observed in either case. This rules out the hypothesis put forward by Bayman and Tang for the anomalous behaviour of $Z = 2$ fragments from $^{12}$C-Em collisions.

To summarize our results on $Z = 2$ fragments we present in Table 6.9 the values of the mean free path for $D \leq 3$ cm and $D > 3$ cm, where $D$ is the distance from the interaction vertex. The values of the mean free path for $Z = 2$ fragments in the two intervals of distance are the same within statistical errors. Thus the behaviour of $Z = 2$ fragments from $^{12}$C-Em collisions at 4.5 A GeV/c is found to be the same as the primary beam of the He nuclei. The average value of the mean free path observed in the present experiment is $(20.09 \pm 0.79)$ cm which compares well with $(20.83 \pm 0.52)$ cm observed by El-Nadi et al (71) for the primary He nuclei at 2.1 A GeV/c.

Thus, we do not find any evidence for anomalously shorter mean free path for the $Z = 2$ projectile fragments in the first few centimeters of the production point. Our results are not in agreement with the results recently obtained by Ghosh et al (72) and El-Nadi et al (48) at the same incident momentum.
Fig. 6.11 Mean free path versus distance from the production point for $Z = 2$ fragments emitted in different reaction channels. The dashed lines represent the average value.
Table 6.9

Mean free paths at different distances from the production point for different categories of the Z = 2 fragments (Ni is the number of collisions)

<table>
<thead>
<tr>
<th>Type of collisions</th>
<th>N1</th>
<th>(D \leq 3) cm</th>
<th>(\lambda) (cm)</th>
<th>(\chi^2/\text{D.O.F.})</th>
<th>N1</th>
<th>(D &gt; 3) cm</th>
<th>(\lambda) (cm)</th>
<th>(\chi^2/\text{D.O.F.})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_h \geq 0)</td>
<td>208</td>
<td>4337.07</td>
<td>20.85 ± 1.45</td>
<td>0.35</td>
<td>327</td>
<td>6545.97</td>
<td>20.02 ± 1.11</td>
<td>0.29</td>
</tr>
<tr>
<td>(N_h \leq 1)</td>
<td>76</td>
<td>1739.99</td>
<td>22.89 ± 2.63</td>
<td>0.65</td>
<td>137</td>
<td>2639.24</td>
<td>19.26 ± 1.64</td>
<td>0.25</td>
</tr>
<tr>
<td>(N_h &gt; 1)</td>
<td>132</td>
<td>2597.08</td>
<td>19.67 ± 1.71</td>
<td>0.36</td>
<td>190</td>
<td>3906.73</td>
<td>20.56 ± 1.49</td>
<td>0.46</td>
</tr>
<tr>
<td>(\Theta \leq 1)</td>
<td>157</td>
<td>3319.90</td>
<td>21.15 ± 1.69</td>
<td>0.62</td>
<td>246</td>
<td>5134.50</td>
<td>20.87 ± 1.33</td>
<td>0.53</td>
</tr>
<tr>
<td>(\Theta &gt; 1)</td>
<td>51</td>
<td>1017.17</td>
<td>19.94 ± 2.79</td>
<td>0.08</td>
<td>81</td>
<td>1411.47</td>
<td>17.43 ± 1.94</td>
<td>0.70</td>
</tr>
<tr>
<td>3 x He</td>
<td>24</td>
<td>501.36</td>
<td>20.89 ± 3.42</td>
<td>0.36</td>
<td>56</td>
<td>1179.36</td>
<td>21.06 ± 2.78</td>
<td>0.51</td>
</tr>
<tr>
<td>2 x He</td>
<td>95</td>
<td>2164.95</td>
<td>22.79 ± 2.39</td>
<td>0.95</td>
<td>157</td>
<td>3296.52</td>
<td>21.00 ± 1.71</td>
<td>0.26</td>
</tr>
<tr>
<td>1 x He</td>
<td>74</td>
<td>1340.17</td>
<td>18.11 ± 2.16</td>
<td>0.32</td>
<td>110</td>
<td>2016.18</td>
<td>18.33 ± 1.70</td>
<td>0.82</td>
</tr>
</tbody>
</table>
6.7 Conclusions

The following conclusions can be drawn from the results presented in this chapter.

(i) The average multiplicities of projectile fragments have a weak dependence on the mass of the target.

(ii) The cross-section for the reaction in which projectile $^{12}\text{C}$ nucleus breaks up into two $\text{Li}$ fragments is < 7.7 x $10^{-4}$ of the total inelastic cross-section.

(iii) The principle of projectile fragmentation observed in electronic experiments does not hold under the condition of $4\pi$-geometry. It means that the fragmentation of the projectile nucleus cannot be described in terms of the participant-spectator model.

(iv) The average multiplicities of fragments of all charges are found to increase with the mass of the projectile and the dependence can well be described by a relation of type: \[ \langle N_Z \rangle = \text{Const. } A^\alpha \]

(v) The angular distributions of the projectile fragments are typically narrow and their dispersions decrease with increasing fragment charge $Z$.

(vi) The properties of the emission of projectile fragments remain strikingly independent of target in peripheral collisions.

(vii) The emission frequency of light as well as heavy projectile fragments are well described by the collision geometry.

(viii) Presence of large $p_t$ particles distorts the transverse momentum distributions. However, for $p_t \leq 500$ MeV/c, the distributions agree with the predictions of the fragmentation model.
(ix) The values of the nuclear Fermi momentum calculated from the observed values of $\sigma(p)$ are in agreement with that obtained in electron scattering experiment.

(x) The observed excitation energy is of the order of binding energy per nucleon indicating that little energy transfer takes place during the fragmentation of $^{12}\text{C}$ nuclei.

(xi) Their exists not large, but statistically significant azimuthal correlations amongst the projectile fragments of $^{12}\text{C}$. This indicates that the fragmenting nucleus gets a transverse momentum during the collision.

(xii) There is no evidence for anomalously shorter mean free path for the $Z = 2$ projectile fragments in the first few centimeters of the production point.

(xiii) No evidence is found for the production of $^6\text{He} + ^6\text{Be}$ binary cluster system as suggested by Bayman and Tang recently.
REFERENCES


LIST OF PUBLICATIONS

Research Papers Published:


Research Papers Presented at Scientific Conference:

10. Correlations in P-lucite and $\alpha$-lucite interactions at cosmic ray energies. 74th Session of the Indian Science Congress Association held at Bangalore (India) from Jan. 3-8, 1987.
12. Mean free path of $\alpha$-particles produced in $^{12}\text{C}$-Em interactions at 4.5 A GeV/c. 75th Session of the I.S.C.A. held at Pune (India) from Jan. 7-12, 1988.
Interactions of Relativistic Carbon Nuclei in
Nuclear Emulsion.

M. Q. R. Khan, M. S. Ahmad, K. A. Siddiqui and R. Hasan
Department of Physics, Aligarh Muslim University - Aligarh-202001, India

(ricevuto il 22 Maggio 1987)

Summary. — A sample of 2200 events is used to study the general
characteristics of \(^{12}\)C-Em interactions at 4.5 A GeV/c. Multiplicity and
angular distributions of charged secondary particles and correlations among
them are discussed. The presented data are compared with the
corresponding results from interactions of other projectiles. Multiplicities of
projectile fragments in different target ensembles of \(^{12}\)C-Em interactions are
studied. The results indicate the violation of the principle of fragmentation.
The dependence of the multiplicities of projectile fragments on the mass of
the projectile is also investigated. It is found that the multiplicities of
fragments of all charges increase with the mass of the projectile and the
dependence can be described by the relation \(N_s = \text{const}A^\alpha\).

PACS. 29.40. - Radiation detectors.

1. - Introduction.

The first experimental evidence of the presence of heavy nuclei in cosmic rays
was reported by Frier et al. (1). This marked the beginning of a new phase in the
investigation of high-energy nuclear interactions, namely the study of nucleus-
ucleus interactions. However, extensive investigations could not be done due to
low statistics and large uncertainties in the estimation of charge and energy of
heavy nuclei in cosmic rays. The availability of relativistic nuclear beams at
Berkeley and Dubna made it possible to study various aspects of nucleus-nucleus
interactions at high energies. A study of nucleus-nucleus interactions may help
in refining the models of multiparticle production in hadron-hadron and hadron-
nucleus interactions. One may get information about the behaviour of nucleons
interacting collectively (24) and also study the behaviour of nuclear matter under
extreme conditions (2).
We have the pleasure to inform you that your paper
"Interaction Mean Free Path of Projectile Fragments
from $^{12}$C-Euqulation Collisions at 4.5 A GeV/c."
has been accepted for publication in "Il Nuovo Cimento" Sect. A, and sent today to the Editorial Office.

To speed up publication you can renounce to correcting the proofs. Should you wish to do so, please return the other part of this postcard at your earliest convenience and possibly within 10 days of receipt.

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