DETERMINATION OF BEND LOSS COEFFICIENT OF THE HYDRAULICALLY SMOOTH PIPE BENDS

DISSERTATION SUBMITTED
IN PARTIAL FULFILMENT OF THE REQUIREMENT
FOR THE DEGREE OF
Master of Science
IN
MECHANICAL ENGINEERING

BY
Kaushal Kumar

Under the able guidance of
Mr. S. M. YUSUF
READER

DEPARTMENT OF MECHANICAL ENGINEERING
Z. H. COLLEGE OF ENGINEERING & TECHNOLOGY
ALIGARH MUSLIM UNIVERSITY
ALIGARH, 1981
DETERMINATION OF BEND LOSS COEFFICIENT OF HYDRAULICALLY SMOOTH PIPE BENDS.

DISSERTATION

( KAUSHAL KUMAR )
M.Sc. Engineering
A.I.I., ALIGARH
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CERTIFIED that the dissertation entitled "DETERMINATION OF BEND LOSS COEFFICIENT OF THE HYDRAULICALLY SMOOTH PIPE RINGS" which is being submitted by Mr. Kamal Kumar in partial fulfilment for the award of the degree of Master of Science in Mechanical Engineering is a record of bonafide work carried out by him under my guidance and supervision. The matter presented here has not been submitted for the award of any other degree or diploma.

(S. K. RASUV)
READER
MECHANICAL ENGINEERING DEPT.,
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ACKNOWLEDGEMENT

WITH a deep sense of gratitude, I thank Mr. S.N. Mumtaz, Reader, Mechanical Engineering Department, Aligarh Muslim University, Aligarh for his constant encouragement and effective and able guidance, rendered during the execution of present work.

I am also thankful to Prof. J.A. Muir, Head of the Mechanical Engineering Department for providing me necessary facilities.

(KAUSHAL KUMAR)
SUMMARY

The flow of fluids in bends has been analysed by many investigators like Yamell, White Adler, Dean Cumings, Beeker, Poule, Kar, Nagler, Ito, Hofmann, Kenlygan, Beij, Abramovich and others. The previous investigators formulated several experimental and analytical equations for the loss of head and loss coefficient for various types of bends. Here the efforts are made to give a semi-analytical equation for the bend loss coefficient of a hydraulically smooth pipe bends loss on the work of previous investigators. The equations predict the total bend loss coefficient in hydraulically smooth pipe bends of varying bend angles and curvatures. For this purpose the help of dimensional analysis has been utilised keeping in view of the previous work. These equations can be modified for other pipe bends having different wall roughness after changing the value of pipe resistance coefficient \( \lambda_0 \).

As the nature of fluid flow in pipe bends is usually turbulent, the bend loss coefficient in pipe bends are formulated for turbulent flows (Reynold's number \( R_e \) ranging from \( 5 \times 10^3 \) to \( 1 \times 10^7 \)).
NOTATIONS

\( h \) = Bend loss of head.
\( R_e \) = Reynold's number
\( R \) = Radius of curvature of pipe bend.
\( r_o \) = Radius of pipe bend.
\( \frac{R}{r_o} \) = Relative radius of curvature of the pipe bend.
\( \lambda \) = Frictional resistance coefficient of pipe bend.
\( \lambda_o \) = Frictional resistance coefficient of straight pipe.
\( v \) = Average velocity of flow through the pipe bend.
\( \beta \) = Bend angle
\( \alpha \) = Kinetic energy transport coefficient.
\( \beta \) = Momentum transport coefficient.
\( \nu \) = Kinematic viscosity of the flowing fluid.
\( a_o \) = Area of cross section of pipe bend.
\( Q_a \) = Actual volume rate of flow.
\( K \) = Total bend loss coefficient.
\( K_b \) = Bend loss coefficient due to change in the direction of the flow in the bend.
\( K_f \) = Bend loss coefficient due to fluid friction in the bend.
\( n \) = Exponent in the Blasius velocity distribution equation for turbulent flow.
\( g \) = Acceleration due to gravity.
CHAPTER I

1.1: INTRODUCTION

Transportation of fluids through pipes and bends is frequently dealt with by Engineers. Distribution of water and gas to domestic consumers through conduits, the supply of steam through pipes in thermal power/plants and of gases in process plants, off-shore pumping of oil etc., are some of the examples of transportation of fluids through pipes and bends. In order that the designer of such systems may provide for adequate pumping requirements, it is necessary to study the loss of head characteristics for fully developed flow through pipes and pipe bends. The prediction of the loss of head in pipes and pipe bends, enables the designer to estimate the power consumption and hence the type and size of pumps required for a given application.
Whenever the fluid flows through a pipe which has a curved portion, it will be subjected to additional loss of energy due to secondary flow. It is well known fact that whenever there is a bending of stream lines some centrifugal forces appear and are directed radially outwards. As a result of this centrifugal force, the pressure in the stream increases along the radius of curvature and velocity correspondingly falls. For this reason, there always occur on velocity redistribution before and after the bend as shown in the figure (A). The stream lines of the flow in bends are not only curvilinear but also interwoven, resulting in spiral currents and crosswaves. The variation in the velocity distribution brought out by the bend will result in boundary separation and formation of zones of eddies. The size of these zones of eddies obviously depends on the curvature of the bend and the velocity of the flow. The influence of relative radius of curvature is more in case of laminar flow than in turbulent flow. The zones of eddies formed near the outer wall of bend is of negligible size. However, the eddies formed near the inner wall are very intense and their zone extends to a considerable distance down stream of the bend.

Even after the main stream fills the pipe a considerable length of straight pipe is required to attain the normal pipe flow condition (fully developed flow). The formation of zones of eddies are the reasons for the loss of energy as the flow takes place through the bend.

The study of losses in bends was theoretically investigated by many investigators assuming to potential flow
Reddy & Kar \cite{1} established a modified theoretical equation for the loss of head in smooth pipe bends having a bend angle ($\theta_b$). The bend loss of head as formulated by Reddy & Kar is given by:

$$h = k_b \frac{v^2}{2g}$$

Where,

$$k_b = 2 \beta (R/r_o) \left[ 1 - \cos \left( \frac{\theta_b}{R/r_o} \right) \right]$$

$$\beta = \frac{(1 + n)^2 (1 + 2n)^2}{2n^2 (2 + n) (2 + 2n)}$$

for turbulent flow with velocity distribution

$$\frac{v}{v_{\text{max}}} = (1 - r/r_o)^{1/n}$$

Where

$$n = f \left( Re \right)$$ determined experimentally by Nikuradse \cite{2}.
TABLE FOR $R_e$, $n$ 

<table>
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<tr>
<th>S.No.</th>
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<th>$\lambda_0$ Due to Prandtl</th>
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This equation does not take into account the loss of head due to pipe friction. Thus, in calculating the total losses, the loss of head due to friction in bend pipe should be included. The frictional loss of head is pipe bend, can be estimated using wall known Darcy's Weisbach equation.

$$h_f = \frac{\lambda_1 b v^2}{d_0 2 g}$$

Thus, the total loss of head due to bend is the sum of the loss of head due to change in direction of flow and the loss of head due to friction in pipe bend.
Fig. (A). Flow in curved pipe after Prandle (35)
CHAPTER - II

2.1: GENERAL REMARK

The motion of fluid particles under turbulent flow conditions are very disorderly and it is not possible to assign any set pattern to their movement. For this reason, the turbulent flow can neither be steady nor uniform, because for such classification instantaneous velocities of fluid particles are to be considered. The motion of a stream having such a structure can not be studied and analysed. Bussenisque has suggested a means of simplifying the structure of the stream so that it can be analysed mathematically.

Bussenisque suggested that in place of the instantaneous actual velocity of the fluid particles passing through a given point in space, the average velocity at the point in space at a suitable interval of time may be taken. If the average value of the velocity at a given point remains constant with time, the flow is said to be steady where as the flow is said to be uniform if at a given instant this average value of the velocity is constant at all points of the stream.

Obviously in case of the flow of real fluids, uniform flow cannot exist due to no slip condition of the flow. Thus if we consider the case of laminar flow of a fluid through a circular pipe, the velocity distribution along the diameter will be parabolic. However, in case of turbulent flow, the maximum velocity changes only within a thin layer of the stream close to the boundary. Thus, in case of turbulent flow of a fluid through a uniform conduit, the main core of
the stream will have more or less uniform flow and it is customary to assume such a flow to be uniform.

When a fluid flows through a pipe, it experiences a certain amount of loss of energy. This loss of energy per unit weight of the fluid flowing is known as the loss of head. The loss of head through in a pipe is essentially due to the following two factors.

i) The viscous forces because of the velocity gradient in the fluid flow field & molecular exchange.

ii) The resistive forces due to molecular exchange of fluid particles between adjacent fluid layers because of the pulsations in the stream.

The viscous forces are proportional to the mean velocity of flow of the stream whereas the forces due to molecular exchange are proportional to the square of the mean velocity of flow.

Flow through a pipe may be either laminar or turbulent depending upon the relative predominance of the viscous or inertia forces. A convenient measure of the relative importance of these two forces is the Reynold’s number, defined as:

$$Re = \frac{\nu d_0}{\nu}$$

Where,

- $\nu$ = Average velocity of flow in a pipe bend.
- $d_0$ = Diameter of pipe bend.
- $\nu$ = Kinematic viscosity of the fluid flowing.

Under normal operating conditions, the flow is laminar for $Re < 2000$ and turbulent for $Re > 3000$. A transition zone exists in the intermediate range. In case
of fluid stream under laminar flow conditions, pulsations in the motion of the fluid particles are completely absent and all the loss of head is only due to viscous forces. Thus, for laminar flow, the loss of head is proportional to the mean velocity of flow.

In case of fluid flow with intense turbulence, the resistive forces due to molecular exchange are much more as compared to viscous forces and thus in case of turbulent flow the loss of head will be proportional to the square of the mean velocity of flow of the stream. If the turbulent in the stream is not very intense, both the viscous forces as well as the resistive forces due to the molecular exchange play their role.

In case of pipe bends, the loss of head is mainly due to the direction change of flow in addition to the loss of head due to pipe friction. This loss of head due to direction change depends upon the relative radius of curvature ($R/r_o$) and bond angle ($\theta_b$).

2.2: THEOREY

The loss of head due to the change in direction of the fluid flow in bends can be estimated using the Kar & Villemonte equation based on the transport coefficients $'\alpha'$ & $'B'$. Kar & Villemonte formulated the following equation for bends of uniform cross-sectional area and small bend angles.

$$K_b = 2B \left[ 1 - \cos (\Delta \theta) \right]$$

For bends with large bend angle $\theta_b$, the bend
is divided into small bend elements of bends angle

\[ \Delta \theta = \frac{\theta_b}{(R/r_o)} \] each as shown in figure (B) and the loss of head can be computed summing them all. Thus,

\[ h_b = 2 \beta \left[ 1 - \cos \left( \frac{\theta_b}{R} \right) \right] \left( \frac{R}{r_o} \right) \]

The loss of head due to friction can be computed using the Darcy's Weisbach equation.

\[ h_f = \frac{\lambda b}{d_o} \cdot \frac{v^2}{2g} \]

\[ = \frac{\lambda b}{(2r_o)} \cdot \frac{v^2}{2g} \]

The resistance coefficient \( \lambda \) for turbulent flow in a curved pipe with the Blasius velocity distribution

\[ \frac{v}{v_{\text{max}}} = (1 - r/r_o)^{1/7} \], can be estimated using White's equation as given below.

\[ \lambda = \lambda_0 \left[ 1 + \frac{3}{40} \frac{R_e^2}{(r_o/R)^{1/2}} \right] \]

where,

\[ \lambda_0 \] is the resistance coefficient for smooth pipe due to Prandtl[ ] as

\[ \frac{1}{\sqrt{\lambda_0}} = 2 \log_{10} \left( R_e \sqrt{\lambda_0} \right) - 0.8 \]

which can be modified for general turbulent flow velocity distribution

\[ \frac{v}{v_{\text{max}}} = (1 - r/r_o)^{1/n} \]

\[ \lambda = \lambda_0 \left[ 1 + \frac{3}{40} R_e^{2.5} (r_o/R)^{1/2} \right] \]

where

\[ \lambda_0 \] is the resistance coefficient for smooth pipe due to Prandtl[ ] as
\[
\frac{1}{\sqrt{\lambda_0}} = 2 \log_{10} \left( R_e \sqrt{\lambda_0} \right) - 0.8
\]

As the total loss of head \( h_f \) is due to the change in the direction of the fluid flow \( h_b \) and resistive force \( h^f \).

The total loss of head equation can be expressed as

\[
h = h_b + h_f = (K_b + K_f) \frac{v^2}{2g}
\]

\[
h = K \frac{v^2}{2g} = K_b \frac{v^2}{2g} + \frac{\lambda_b}{\lambda_0} \frac{v^2}{2g}
\]

The total bend loss coefficient can be expressed as:

\[
K = K_b + K_f
\]

\[
K = 2 B \left( \frac{R}{r_o} \right) \left[ 1 - \cos \left( \frac{\theta_o}{R} \right) \right] + \lambda_0 \left[ \frac{1}{2} \frac{R}{r_o} \frac{r_o}{(r_o/R)^{1/2}} \right] \cdot \frac{R \theta_o}{2 r_o}
\]

where,

\[
B = \frac{(1 + n)^2 (1 + 2n)^2}{2n^2 (2 + n) (2 + 2n)}
\]

and

\[
\frac{1}{\sqrt{\lambda_0}} = 2 \log_{10} \left( R_e \sqrt{\lambda_0} \right) - 0.8
\]

In this present work we have calculated the bend loss coefficient due to curvature and friction for different \( R/r_o \) ratios, different Reynold's numbers and different bend angles. The range of various parameters considered are as follows:

\[
\frac{R}{r_o} = \pi, 2, 4, 6, 8, 12, 16, 20
\]
\[ R_e = 5 \times 10^3, \; 1 \times 10^4, \; 2 \times 10^4, \; 5 \times 10^4, \; 1 \times 10^5, \]
\[ 2 \times 10^5, \; 5 \times 10^5, \; 1 \times 10^6, \; 2 \times 10^6, \; 5 \times 10^6, \]
\[ 1 \times 10^7. \]

\[ \theta_b = 30^\circ, \; 45^\circ, \; 60^\circ, \; 90^\circ, \; 120^\circ, \; 150^\circ. \]
FIG. (5) SMOOTH BEND
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CHAPTER 3

3.1 RES: LTS & CONCLUSIONS

Analytical estimation of the bend loss coefficient (K) which takes into account of the pipe friction losses and effect of bend curvature are illustrated in Fig. 1 to 37 for radius ratios (R/ro) ranging from 2 to 30 and bend angles ranging from 30° to 150°, under turbulent flow conditions. From the estimated results of present work, the following facts can be concluded.

(1) As illustrated in Fig 1 to 7c (KVs G for R/ro = Constt), the bend loss coefficient (K) increases with an increase in bend angles for a fixed "Re". As "Re" increases the bend loss coefficient decreases.

(2) As illustrated in Fig 8 to 13c (KVs R/ro for Re=Constt), the bend loss coefficient (K) first decreases at low radius ratios (R/ro) and then it increases at higher radius ratios (R/ro). As the "Re" increases, the increase in the bend loss coefficient at higher radius ratios is not very significant.

(3) From Fig 14 to 24c (KVs R/ro for Re=Constt), the bend loss coefficient (K) first decreases at low radius ratios (R/ro) and then it increases gradually at higher radius ratios. For different bend angles (θ) and at different "Re", the optimum radius ratios have been indicated with the dotted lines. As the bend angles or "Re" increases, the optimum (at which the bend loss coefficient (K) in minimum) radius ratio also increases.

(4) From the Fig 25 to 30c (KVs Re for G= Constt), the bend loss coefficient (K) decreases as the radius ratio (R/ro) increases and then beyond a certain value of radius ratio (which is optimum radius ratio), the bend loss coefficient increases. At different bend angles (θ), the value of optimum radius ratio (R/ro) at low & high Reynolds number are as follows:
The above table indicates that the bend loss coefficient reduces to a minimum if the radius ratio and bend angle are increased. The value of the optimum radius ratio is higher at high "Re" as shown in the above table. From the above table, one can easily obtain the value of the optimum radius ratio for different bend angles and at various flow conditions.

(8) From fig 31 to 37, (Kvs Re at R/ro = const) indicate that at larger bend angles, the bend loss Coefficient (K) is high. As the "Re" increases, the bend loss Coefficient (K) decreases. At high radius ratios (R/ro), the difference in the bend loss Coefficient (K) is more. This difference becomes less as the value of the bend angle or / and the radius ratio (R/ro) decreases.

The above analysis clearly indicates that the bend loss Coefficient (K) depends mainly on the radius ratio (R/ro), the bend angle (θ) and "Re".

\[ K = f(R/ro, θ, Re) \]

The above analysis can be extended if the roughness of the pipe is taken into account. The fluid (of different viscosities) flowing through the bends will also affect the bends loss Coefficient. A similar study can be made experimentally. The results obtained analytically can be verified experimentally. A modified empirical equation can be developed using experimental results in the light of the present analytical investigation.
FIG (2.5) BEND LOSS COEFFICIENT OF 30° BEND AGAINST REYNOLDS NUMBER FOR DIFFERENT RADIUS RATIO
FIG. (26) BEND LOSS COEFFICIENT OF 45° BEND AGAINST REYNOLDS NUMBER FOR DIFFERENT RADIUS RATIO
Fig (27) Bend loss coefficient of 60° bend against Reynolds number for different radius ratio.
Fig (2.8) Bend loss coefficient of 90° bend against Reynolds number for different radius ratios.
Figure 29: Bend Loss Coefficient of 120° Bend Against Reynolds Number for Different Radius Ratio.
Bend loss coefficient of 150° bend against Reynolds number for different radius ratio.
Fig. (21) Bend Loss Coefficient against Reynolds Number for different Bend Angles.
\[
\frac{R}{\rho} = 4.
\]

**Figure 32** Bend Loss Coefficient Against Reynolds Number for Different Bend Angles
FIG. 33. BEND LOSS COEFFICIENT AGAINST REYNOLDS NUMBER FOR DIFFERENT BEND ANGLES.
Figure C34: Bend loss coefficient against Reynolds number for different angles.
Figure 3.3 Bend Loss Coefficient Against Reynolds Number for Different Bend Angles

\( \frac{R}{f_c} = 12 \)

\( \theta = 150^\circ, 120^\circ, 90^\circ, 60^\circ, 45^\circ, 30^\circ \)
FIG. (36) BEND LOSS COEFFICIENT AGAINST REYNOLDS NUMBER FOR DIFFERENT BEND ANGLES
FIG. (37) BEND LOSS COEFFICIENT AGAINST REYNOLDS NUMBER FOR DIFFERENT BEND ANGLES.
FIGURE VARIATION OF \( \eta \) AGAINST REYNOLDS NUMBERS
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